$$\begin{cases} a_i = \dfrac{(n+1-i)(bb_{i-1}-aa_{i-1})}{a^2+b^2} \\ b_i = \dfrac{-(n+1-i)(ab_{i-1}+ba_{i-1})}{a^2+b^2} \\ i=1,2,\cdots,n \\ a_0 = -\dfrac{b}{a^2+b^2},\ b_0 = \dfrac{a}{a^2+b^2} \end{cases}$$

注:当 $n=0$ 时,由(25),(26)式可得

$$\int e^{ax}\cos bx\,\mathrm{d}x = \frac{a\cos bx + b\sin bx}{a^2+b^2}e^{ax} + c$$

$$\int e^{ax}\sin bx\,\mathrm{d}x = \frac{a\sin bx - b\cos bx}{a^2+b^2}e^{ax} + c$$

注意上述两个积分公式用例 8 的方法直接求也很容易.

当 $n=1$ 时,由(25),可得

$$\int x e^{ax}\cos bx\,\mathrm{d}x = \frac{a\cos bx + b\sin bx}{a^2+b^2}xe^{ax} + \frac{(b^2-a^2)\cos bx - 2ab\sin bx}{(a^2+b^2)^2}e^{ax} + c$$

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A Transformation in Ordinary Differential Equations

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Abstract

A transformation recommended in the teaching of ordinary differential equations to help us find the particular solutions of both nonhomogeneous linear differential equations with constant coefficients and nonhomogeneous Euler's equations. Its applications to integrals is also discussed.

Keywords transformation; particular solution; integral