ON THE REALISTIC STRESS SPACE OF SOLIDS

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ABSTRACT

Solids can not be infinitely stressed under all stress states, thus only certain portion of the stress space has significance for solids. The shape, size and structure of this portion of stress space is closely connected with the mechanical properties of solids. By "realistic stress space of solids", is meant this portion of the stress space. Although some aspects of this problem had been investigated by some authors, there exists even no definite concept regarding the shape, structure and properties of the realistic stress space of solids up to this date.

The conclusions obtained by the author in a previous paper[1] provides a possibility to make a first study on this problem. This is done by connecting these conclusions with Von Mise's equation of plasticity which can be written in any Cartesian system with the hydrostatic axis as s_z-axis:

$$\frac{s_x^2}{\left(\sqrt{\frac{2}{3}}K\right)^2}+\frac{s_y^2}{\left(\sqrt{\frac{2}{3}}K\right)^2}=1. \tag{1}$$

The isothermal surface of rupture obtained in the previous paper can be written in the same system as:

$$\frac{s_x^2}{\left(\sqrt{\frac{2E\alpha}{(1+\mu)r_0}}\right)^2}+\frac{s_y^2}{\left(\sqrt{\frac{2E\alpha}{(1+\mu)r_0}}\right)^2}\pm\frac{s_z^2}{\left(\sqrt{\frac{2E\alpha}{(1-2\mu)r_0}}\right)^2}=1,\left\{\begin{matrix}(+)\text{ for }\theta\geqslant 0,\\(-)\text{ for }\theta\leqslant 0,\end{matrix}\right\} \tag{2}$$

where

E=Young's modulus of elasticity;
α=surface free energy per unit area of the existing crack;
r_0=radious of the existing crack;
K=yielding point of the solid in uni-axial stress;

$$\theta=\frac{1}{3}(s_1+s_2+s_3)$$

s_1,s_2,s_3=principal stresses;
μ=Poisson's ratio.

The yielding surface (1) is a cylinder with the hydrostatic axis as long axis, and the effective parts of the surface of rupture form a bell-like surface of revolution co-axial with the cylinder, termed as "the bell surface of rupture". The bell surface intersects with the cylinder in a circle, the portion of the bell surface above the intersecting circle is termed as "the bell-crown."

Besides, there are still another three surfaces of significance, that is, the brittle cone which is the cone passing through the base circle of the bell crown; the plane of pure shear which is the plane $\theta=0$ and the non-fracturing cone which is the asymptotic cone of the lower portion of the bell surface.

The question as to how far the realistic stress space extends in the octant of pure compression can only be qualitatively answered at present. Because infinite process of deformation under infinitely growing pressure could occur only if the acting instruments were absolutely rigid bodies, the realistic stress space should be bounded by some not yet known surface defined by the mutual relation between the deforming and the deformed bodies.

The above surfaces are the limiting surfaces of the realistic stress space of solids. The significance of this space becomes obvious when various processes of deformation are considered in connection with it. A process of deformation under constant stress state is described by a position vector R of a point P in the realistic stress space:

$$R = \overrightarrow{OP} = [s_1,\ s_2,\ s_3] = \overrightarrow{\sigma} + \overrightarrow{t}, \tag{3}$$

where,

t=shear component of R,
σ=hydrostatic component of R.

R bears the following significance:

1) it describes a process of deformation, $\overrightarrow{OP}$;
2) it gives the stress states at the end of the process, $[s_1, s_2, s_3]$;
3) it shows that a given process of deformation is composed of a shear deformation and an elastic volume change; and
4) it shows that $\overrightarrow{\sigma}$ is responsible only for volume change and $\overrightarrow{t}$ is responsible only for shear deformation.

With the above concepts, it is possible to present a formal interpretation to the realistic stress space of solids as follows:

The realistic stress space of soldis is bell-shaped, closed, and symmetrical to the hydrostatic axis, and is composed of the following parts:

1) The state of the lowest potential energy: the origin, which is the starting point of the process of deformation;

2) The elastic cyclinder: the portion of Von Mise's cylinder under the bell crown and above the unknown limiting surface in the octant of pure compression;
3) The plastic region: all the stress states between the elastic cylinder and the bell surface of rupture;
4) The bell-surface of rupture: all the stress states under which rupture occurs in solids; and
5) The non-fracturing limiting surface to be defined by the mutual relation between the deforming and the deformed bodies.

It is understood that elastic and plastic deformation of solids occurs under the stress states in the above mentioned elastic and plastic regions.

The above picture agrees with the well-known facts and physical principles regarding the strength properties of solids, and brings out the concept of "factor of plasticity" $\triangle t$:

$$\triangle t = t_f - t_0 , \tag{4}$$

where t_f is the value of t at the bell surface of rupture, and t_0 is the radius of Von Mise's cylinder. Because Δt is responsible only for plastic deformation, and for a given solid under a given temperature and a given strain rate, the ductility of the solid is measured by Δt and the rate of strain hardening which is independent of stress states according to the modern theory of strength, thus Δt can be termed as "factor of plasticity". By this concept, the ductility of solids under various stress states can be predicted by the central section of the bell of stress space, and the predictions agree with the well-known facts on of ductility of solids under various stress states.

According to the above concepts, the processes of deformation should be logically classified as follows:

1) Brittle deformation: any vectorial process of deformation within the brittle cone does not intersect with the yielding cylinder, but directly ends on the bell crown; this leads to brittle rupture.
2) Ductile deformation ending rupture: any vectorial process of deformation between the brittle cone and the non-fracturing cone intersects both the yielding cylinder and the bell surface of rupture, and ends with definite amount of ductility defined by the corresponding value of the factor of plasticity. Such processes can be subdivided into three groups:
    a. ductile tensile deformation,
    b. shear deformation,
    c. compressive deformation.
3) Non-fracturing process of deformation: all vectorial process of deformation within the non-fracturing cone fail to interset with the bell surface of rupture, in such processes of deformation, a solid can undergo as much plastic deformation or elastic deformation or volume change as the deforming instrument permits.

All technical processes of deformation find their vectorial path in the realistic stress space, such processes are as material testing, shearing operations, and the most important of all, plastic forming of metals. In the last group of deformation processes, metals had been found to be able to undergo deformation without the possibility of rupture in some cases; it is the theory of this paper that provides a formal interpretation to this phenomenon.

In conclusion, it can be stated:

That the present theory provides the first definite and clear concept on the shape, structure and properties of the realistic stress space of solids; and

That the predictions of the present theory agree with the well-known concepts and facts regarding the strength properties of solids.