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# Boundary value analysis of parallel plate capacitors*

ZHAO Lin, JIANG Ze

(Chongqing University of Posts and Telecommunications, Chongqing 400065, P. R. China)

**Abstract**: By taking the computation of capacitance of a parallel plate capacitor as a boundary value problem, a formula for the computation with any ratio of the plate separation to the radius of the plate is presented. The model shows effectiveness by the good agreement between the analytical and the numerical results.

**Key words**: plate capacitor; capacitance; boundary value problem

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## 0 Introduction

With the development of circuits and related theories, the analysis and designing of a capacitor, one of the fundamental elements, has been an important subject actively studied by technicians in related areas. For the importance of the problem, the relevant analysis and computation have been fundamental contents of classical electromagnetic theories. Actually, the problem is analyzed by field theory, not only for the internal consistence between field theory and circuit ones, but also for the thorough understanding of physical characteristics of the capacitor, paving the way for optimization design of various applications.

A parallel capacitor is widely used for its simple structure, and it is also the basic model for the analysis of capacitance of capacitors. Furthermore, it has semeiological meaning in circuit theory. Its fringe effect is often neglected for simplifica-tion[1-3], thus apparently, the obtained results are only rough ones. The problem is properly treated with conformal mapping[4], however, it has obvious limits.

In this paper, a parallel plate capacitor is modeled analytically, on which a boundary value problem of the potential field is set, and then the analytical presentation of the capacitor is obtained. The method for analysis also has values for the analysis of capacitors with no plate but geometrically regular structures.

## 1 Structure and analysis of model

As is shown in Fig. 1, the plates of the capacitor consist of two parallel conductor discs, between which the separation is $h$. Assuming the potential difference between the two plates is $V_0$, the equal but opposite sign charge of the two plates is $Q$, then the capacitance between the two-plates is

$$C = \frac{Q}{V_0} \tag{1}$$

The ratio of $Q$ to $V_0$ in Eq. (1) can be computed by methods of integral functions or functional analysis, and the solution can also be described with equivalent differential equations.

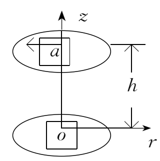

**Fig. 1 A parallel plate cacapacitor**

Furthermore, when the boundary of the field has a geometrically regular structure, a closed analytical solution can be obtained with the latter one, thus theoretical analysis on relevant problems can be taken conveniently. For the consideration, the process is based on analyzing the differential equations on a boundary value problem of a potential field, to obtain calculation relation of the parallel plate capacitor. For this, cylindrical polar coordinates are set as is shown in Fig. 1, hence the

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**Biography**: ZHAO Lin (1969-), female, from Yunnan Province, instructor, interested in the research of electromagnetic field and microwave technology.

boundary value problem can be presented as

$$\begin{cases} \nabla^2 V = 0, & 0 \leqslant r \leqslant a \quad 0 < z < h \\ V|_{z=0} = 0, V|_{z=h} = V_0, & 0 \leqslant r \leqslant a \\ V + \alpha \dfrac{\partial V}{\partial r}\Big|_{r=a} = 0 \end{cases} \tag{2}$$

In Eq. (2), $\alpha$ is coefficient in proportion to a, which describes the functional relation for the radial component of the fringe field. And it is needed that $\alpha|\xrightarrow[a\to\infty]{}\infty$, i.e. the radial component can be neglected then the fringe capacitance can also be neglected.

By separating variables, a formal solution for the boundary value problem in Eq. (2) is obtained

$$V(r,z) = \sum_{n=0}^{\infty} (A_n \mathrm{J}_n(kr) + B_n \mathrm{N}_n(kr)) \begin{Bmatrix} \cos(n\varphi) \\ \sin(n\varphi) \end{Bmatrix} e^{\pm kz} \tag{3}$$

In Eq. (3), $\mathrm{J}_n(kr)$ and $\mathrm{N}_n(kr)$ are $n$-order first kind and second kind Bessel functions, respectively. For the symmetry of the problem, the result should be irrelevant to the coordinate variable $\varphi$. What's more, it is taken into account that $V(r,z)|_{z=0} = 0, V(r,z)|_{r\to 0} < \infty$, therefore, Eq. (3) can be simplified as

$$V(r,z) = \sum_{n=1}^{\infty} A_n \mathrm{J}_0(k_n r)\sinh(k_n z) \tag{4}$$

In Eq. (4), $\sinh(k_n z)$ is a hyperbolic sinusoidal function. $k_n = \dfrac{x_n}{a}$ $(n = 1, 2, 3, \cdots)$, while $x_n$ is a solution of the following equation under the boundary condition where $\rho = a$.

$$\frac{\mathrm{J}_0(x)}{\mathrm{J}_1(x)} = \frac{\alpha}{a} x \tag{5}$$

And in Eq. (4), the coefficient $A_n$ of the Fourier-Bessel series is defined by the boundary condition where $z = h$, i.e.

$$\begin{aligned} A_n &= \frac{1}{\sinh(k_n h)[\mathrm{N}_n^{(0)}]^2} \int_0^a V_0 \mathrm{J}_0(k_n r) r \mathrm{d}r \\ &= \frac{1}{\sinh(k_n h)[\mathrm{N}_n^{(0)}]^2} \frac{a}{k_n} \mathrm{J}_1(k_n a) \end{aligned} \tag{6}$$

In Eq. (6), the normalizing coefficient $[\mathrm{N}_n^{(0)}]^2 = \dfrac{1}{2} a^2 [1 + \dfrac{\alpha^2}{a^2} x_n^2][\mathrm{J}_1(x_n)]^2$. Finally, a solution for the internal potential field of the capacitor is obtained

$$V(r,z) = 2V_0 \sum_{n=1}^{\infty} \frac{1}{(1 + \frac{\alpha^2}{a^2} x_n^2) x_n} \frac{\sinh(\frac{x_n}{a} z)}{\sinh(\frac{x_n}{a} h)} \frac{\mathrm{J}_0(\frac{x_n}{a} r)}{\mathrm{J}_1(x_n)} \tag{7}$$

From Eq. (7), the total charge $Q$ of each plate can be presented

$$\begin{aligned} Q &= \int \sigma_s \mathrm{d}s = \int \varepsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0} \mathrm{d}s = \int_0^{2\pi}\int_0^a \varepsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0} r \mathrm{d}r \mathrm{d}\varphi \\ &= 4\pi\varepsilon_0 V_0 a \sum_{n=1}^{\infty} \frac{1}{(1 + \frac{\alpha^2}{a^2} x_n^2) x_n} \frac{1}{\sinh(\frac{x_n}{a} h)} \end{aligned} \tag{8}$$

Substituting Eq. (8) for Eq. (1), the analytical solution of the capacitance is obtained.

## 2 Numerical simulation

In order to carry out a numerical analysis, $\alpha$ is assumed to be linear with a, and the assumption is reasonable with explanation of the third kind homogenous boundary condition in Eq. (2). When $\dfrac{h}{a} \to 0$, it becomes an ideal parallel plate capacitor, and the corresponding capacitance $C_\infty = \varepsilon_0 \dfrac{S}{h} = \varepsilon_0 \dfrac{\pi a^2}{h}$. The solution $C_a$ given by the paper should equal $C_\infty$, i.e. $\dfrac{C_a}{C_\infty}\Big|_{\frac{h}{a}\to 0} \to 1$. It is shown in Fig. 2 that the requirement can be met, and the validity of the analytical model and related theoretical analysis is clear. It is also shown in Fig. 2, that the relative error of computation using $C_\infty$ is within 10% when $a = 2h$, thus this is useful for computation in engineering.

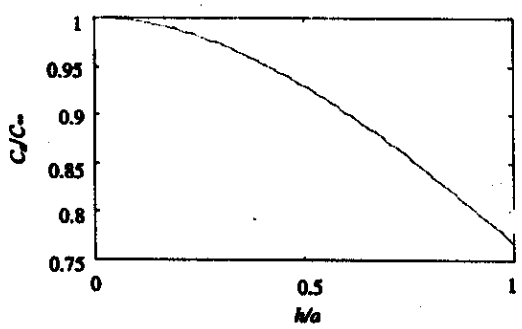


**Fig. 2 Relation between the capacitance and $h/a$**

## 3 Conclusion

In this paper, the computation of capacitance of the parallel plate capacitor is treated as a boundary problem of the field, and it is analyzed and solved with the separation of variables, finally the validity of the result is shown by numerical simulation. The proposed method is useful as theoretical reference for the analysis of capacitors with different structures, furthermore, it shows a principle for the accuracy estimation for the computation of capacitance of capacitors in engineering, thus it has some values in various applications.

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# 平板电容器的边值分析*

赵 琳, 蒋 泽

(重庆邮电学院,重庆, 400065)

**摘　要**:将平板电容器电容的计算作为典型的场边值问题进行处理,从而得到了可适用于对具有任意极板半径与其间隔之比的平板电容器电容的分析求解关系,数值计算结果与有关理论分析的高度一致性,表明了所建立的分析模型的有效性。

**关键词**:平板电容器;电容;边值问题

* 作者简介:赵琳(1969-),女,云南人,讲师,研究方向为电磁场、微波技术。

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# 一种新的 IP DiffServ over OBS 网络体系结构及性能分析*

隆克平[1], Rodney S. Tucker[2], 王重钢[3]

(1. 重庆邮电学院光互联网及无线信息网络研究中心,重庆 400065;
2. 澳大利亚墨尔本大学超宽带信息网络研究中心 ,墨尔本 3010; 3. 香港科技大学)

**摘　要**:IP DiffServ 已经被 IETF 标准化,并由于其实现简单和易于扩展被作为 IP QoS 的一种好的解决办法。提出了一种新的光突发交换网络支持 IP DiffServ 的网络体系结构,称为 DS-OBS,并给出了网络结构、边缘节点和核心节点的功能模型、控制包格式、入口节点的会聚算法和核心节点的调度算法。与目前基于 offset time 的 OBS QoS 结构不同,提出结构的基本思想是:在入口边缘路由器执行业务区分的突发会聚,在核心节点对不同类的控制包执行不同的每一跳行为(PHB)处理,从而实现业务区分。仿真结果表明:提出的结构能在端到端延迟、吞吐量和 IP 分组丢失等对 EF 类、AF 类和 BE 类提供很好的业务区分。

**关键词**:IP 服务质量;区分服务;每一跳行为;光突发交换;突发会聚

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作者简介:隆克平(1968-),男,四川通江人,教授,博士生导师。主要从事光互联网络结构及交换技术、宽带网理论与技术的研究,已发表学术论文 100 余篇,被 SCI、EI 和 ISTP 收录 60 余篇,已申请国际专利 4 项,出版译著 2 部。