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时滞BAM神经网络周期解的存在性和全局指数稳定性*

陈安平
(湘南学院数学系, 郴州 423000)
(E-mail: chenap@263.net)

曹进德
(东南大学数学系, 南京 210096)

黄立宏
(湖南大学数学与计量经济学院, 长沙 410082)

摘　要　本文利用迭合度理论, 通过构造适当的 Lyapunov 泛函并结合 Yang 不等式分析技巧, 获得了具周期系数的时滞 BAM 神经网络周期解的存在性和全局指数稳定性的充分条件, 这些结果对设计全局指数稳定的 BAM 神经网络与周期振荡的 BAM 神经网络具有重要的指导意义.

关键词　时滞 BAM 神经网络; 周期解; 全局指数稳定性; Lyapunov 泛函
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1　引言

最近, [1-3] 提出了一类双向联想记忆神经网络 (BAMNNs), 这类神经网络能储存双极向量对, 并在模式识别、联想记忆、人工智能 [2] 等方面具有广泛的应用, 许多学者研究了具或不具轴突传输时滞的 BAM 神经网络系统的稳定性, 获得了一系列研究成果 [1-21]. 众所周知, 研究神经网络系统的收敛性、振荡性和混沌具有十分重要的理论和实践意义, 目前关于收敛性的研究已经很多且较深入, 但由于神经网络动力系统的复杂性, 振荡和混沌性质的研究十分困难, 发展也很慢. 从神经生物学的观点来看, 神经网络动力系统主要应具有振荡和混沌动力学行为. 然而, 在近二十年的研究成果中, 研究神经网络振荡性的文章较少, 且大多数是在常系数与周期性外部输入的情形下, 获得了一些结果 (见 [4,5,7,8,11,12]). 对于 BAM 神经网络系统, 仅见 [4] 研究了在常系数情形下周期解的存在性和指数稳定性. 本文利用迭合度理论, 通过构造适当的 Lyapunov 泛函, 并结合 Yang 不等式分析技巧 [22] 研究具周期系数的时滞 BAM 神经网络系统周

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期解的存在性和全局指数稳定性，去掉了通常对激励函数的有界性假设.

2 预备知识

考虑具周期系数的时滞 BAM 神经网络系统

$$\dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{p} p_{ji}(t) f_j\big(y_j(t-\tau_{ji}(t))\big) + I_i(t), \tag{2.1a}$$

$$\dot{y}_j(t) = -b_j(t) y_j(t) + \sum_{i=1}^{n} q_{ij}(t) f_i\big(x_i(t-\sigma_{ij}(t))\big) + J_j(t), \tag{2.1b}$$

其中 $i=1,2,\cdots,n$； $j=1,2,\cdots,p$. 变量 $x_i(t)$, $y_j(t)$ 分别表示第 i 个和第 j 个神经元在 t 时刻的状态； $p_{ji}(t)$, $q_{ij}(t)$ 表示在 t 时刻的联接权值； $I_i(t)$ 和 $J_j(t)$ 表示在 t 时刻的外部输入； $\tau_{ji}(t)$ 和 $\sigma_{ij}(t)$ 对应于轴突信号传输时滞.

本文中，我们总假设 a_i 为正常数，函数 $b_j(t)$, $p_{ji}(t)$,$q_{ij}(t)$, $I_i(t)$, $J_j(t)$ 是连续 ω 周期函数且 $b_j(t)>0$, 函数 $\tau_{ji}(t)\geq 0$, $\sigma_{ij}(t)\geq 0$ 是连续可微的 ω 周期函数且 $\dot{\tau}_{ji}(t)<1$, $\dot{\sigma}_{ij}(t)<1$.

系统 (2.1) 的初始条件为

$$x_i(s)=\phi_i(s), \qquad s\in[-\tau,0], \qquad \tau=\max_{1\leq i\leq n;\,1\leq j\leq p}\ \max_{0\leq t\leq\omega}\tau_{ji}(t), \tag{2.2a}$$

$$y_j(s)=\psi_j(s), \qquad s\in[-\sigma,0], \qquad \sigma=\max_{1\leq i\leq n;\,1\leq j\leq p}\ \max_{0\leq t\leq\omega}\sigma_{ij}(t), \tag{2.2b}$$

这里 $\phi_i(s)$, $\psi_j(s)$ 是连续函数.

对系统 (2.1) 的任一解

$$Z(t)=(x,y)^T=\big(x_1(t),x_2(t),\cdots,x_n(t),y_1(t),y_2(t),\cdots,y_p(t)\big)^T$$

和它的周期解

$$Z^*(t)=(x^*,y^*)^T=\big(x_1^*(t),x_2^*(t),\cdots,x_n^*(t),y_1^*(t),y_2^*(t),\cdots,y_p^*(t)\big)^T,$$

定义

$$\big\|(\phi,\psi)^T-(x^*,y^*)^T\big\|_r=\max_{-\tau\leq t\leq 0}\sum_{i=1}^{n}\big|\phi_i(t)-x_i^*(t)\big|^r+\max_{-\sigma\leq t\leq 0}\sum_{j=1}^{p}\big|\psi_j(t)-y_j^*(t)\big|^r,$$

其中 $r>1$ 为常数.

定义 1[23] 矩阵 A =(a_{ij}) 称为非奇异 M 矩阵，如果

(i) $a_{ii}>0$, $i=1,2,\cdots,n$.

(ii) $a_{ij}\leq 0$, 对 $i\neq j$, $i,j=1,2,\cdots,n$.

(iii) $A^{-1}\geq 0$.

定义 2 系统 (2.1) 的周期解 $Z^*(t)$ 称为全局指数稳定的，若对 (2.1), (2.2) 的任何解

$Z(t)$, 存在常数 $\varepsilon > 0$, $M \geq 1$, 使得对一切 $t > 0$ 成立

$$\sum_{i=1}^{n} \left|x_i(t) - x_i^*(t)\right|^r + \sum_{j=1}^{p} \left|y_j(t) - y_j^*(t)\right|^r \\ \leq M \left\|(\phi, \psi)^T - (x^*, y^*)^T\right\|_r e^{-\varepsilon t}.$$

为方便起见，令

$$p_{ji}^+ = \max_{0 \leq t \leq \omega} \left|p_{ji}(t)\right|, \qquad q_{ij}^+ = \max_{0 \leq t \leq \omega} \left|q_{ij}(t)\right|, \qquad I_i^+ = \max_{0 \leq t \leq \omega} \left|I_i(t)\right|,$$

$$J_j^+ = \max_{0 \leq t \leq \omega} \left|J_j(t)\right|, \qquad \widetilde{I}_i = \frac{1}{\omega} \int_0^\omega I_i(t)\,\mathrm{d}t, \qquad k_{ij} = \Big(\max_{0 \leq t \leq \omega} \frac{1}{1 - \dot{\sigma}_{ij}(t)}\Big)^{\frac{1}{2}},$$

$$b_j^- = \min_{0 \leq t \leq \omega} b_j(t), \qquad \dot{\sigma}_{ij}^+ = \max_{0 \leq t \leq \omega} \dot{\sigma}_{ij}(t), \qquad \dot{\tau}_{ji}^+ = \max_{0 \leq t \leq \omega} \dot{\tau}_{ji}(t).$$

假设激励函数 f_k 满足下列条件:

(H1) 存在正实数 $\mu_k > 0$ 使得

$$\left|f_k(x) - f_k(y)\right| \leq \mu_k |x - y|, \qquad f_k(0) = 0,$$

对 $x, y \in R$, $x \neq y$, $k = 1, 2, \cdots, \max\{n, p\}$.

为证明本文的主要结果，我们还需要下列准备工作.

考虑 Banach 空间 X 中的抽象微分方程

$$Lz = \lambda N z, \qquad \lambda \in (0, 1), \tag{2.3}$$

其中 $L : \mathrm{Dom\ L} \cap X \to X$ 是线性算子， λ 为参数. 令 P, Q 为两个投影:

$$P : X \cap \mathrm{Dom\,L} \to \mathrm{Ker\,L} \qquad \text{和} \qquad Q : X \longrightarrow \mathrm{X/ImL}.$$

引理 1[24] 设 X 是 Banach 空间， L 是指标为零的 Fredholm 算子， Ω 是 X 中的有界开集， $N : \overline{\Omega} \to X$ 在 $\overline{\Omega}$ 是 L 紧. 假设

(a) 对任意的 $\lambda \in (0, 1)$, $z \in \partial\Omega \cap \mathrm{Dom\ L}$, $Lz \neq \lambda N z$;

(b) 对任意的 $z \in \partial\Omega \cap \mathrm{Ker\,L}$, $QNz \neq 0$, 且 $\deg\{QNz, \Omega \cap \mathrm{Ker\,L}, 0\} \neq 0$,

则 $Lz = Nz$ 在 $\overline{\Omega}$ 至少存在一个解.

引理 2[23] 假设 A 是非奇异 M 矩阵且 $Aw \leq d$, 则 $w \leq A^{-1}d$.

引理 3 假设 $b_j(v)$ 是周期为 ω 的周期函数， $s : t - \tau_{ji}(t) \to t - \tau_{ji}(t) + \omega$ $(i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, p.)$, 则函数 $K_j(t, s) = \dfrac{e^{\int_t^s b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)\,\mathrm{d}u} - 1}$ 满足不等式 $K_j(t, s) \leq \alpha_j$, 其中 $\alpha_j = \dfrac{e^{\int_0^\omega b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)du} - 1}$.

证 因为 $s : t - \tau_{ji}(t) \to t - \tau_{ji}(t) + \omega$ $(i = 1, 2, \cdots, n;\ j = 1, 2, \cdots, p.)$, $\tau_{ji}(t) > 0$, $b_j(t)$ 为 ω 周期函数，所以

$$0 \leq \int_t^s b_j(v)\,\mathrm{d}v \leq \int_t^{t - \tau_{ji}(t) + \omega} b_j(v)\,\mathrm{d}v \\ = \int_{t - \tau_{ji}(t)}^{t - \tau_{ji}(t) + \omega} b_j(v)\,\mathrm{d}v - \int_{t - \tau_{ji}(t)}^{t} b_j(v)\,\mathrm{d}v \leq \int_0^\omega b_j(v)\,\mathrm{d}v,$$

所以,

$$K_j(t,s)=\frac{e^{\int_t^s b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)\,\mathrm{d}u}-1}\le\frac{e^{\int_0^\omega b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)\,\mathrm{d}u}-1}.$$

3 周期解的存在性

定理 1 假设激励函数 f_k 满足条件 (H1), 又

(i) 矩阵 C 是非奇异 M 矩阵.

(ii) $a_i-\omega\sum\limits_{j=1}^{p}\sum\limits_{l=1}^{n}k_{lj}p_{jl}^{+}\mu_j\alpha_j q_{ij}^{+}\mu_i>0$, 则系统 (2.1) 至少存在一个 ω 周期解.

其中 $C=(c_{il})_{n\times n}$, $c_{il}=a_i\delta_{il}-(1+\omega a_i)c_{il}^{*}$, $c_{il}^{*}=\omega\mu_l\sum\limits_{j=1}^{p}k_{lj}p_{ji}^{+}\mu_j\alpha_j q_{lj}^{+}$, $i,l=1,2,\cdots,n$.

$$\delta_{il}=1,\quad i=l;\qquad \delta_{il}=0,\quad i\ne l,\qquad \alpha_j=\frac{e^{\int_0^\omega b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)\,\mathrm{d}u}-1},\qquad j=1,2,\cdots,p.$$

证 假设 $x_i(t)$ $(i=1,2,\cdots,n.)$ 是一个 ω 周期函数, 我们容易看到方程 (2.1b) 的 ω 周期解是下列系统的 ω 周期解

$$y_j(t)=\int_t^{t+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s,\qquad j=1,2,\cdots,p. \tag{3.1}$$

且

$$\beta_j(s,x)=\sum_{i=1}^{n}q_{ij}(s)f_i(x_i(s-\sigma_{ij}(s)))+J_j(s),\qquad j=1,2,\cdots,p, \tag{3.2}$$

$$K_j(t,s)=\frac{e^{\int_t^s b_j(v)\,\mathrm{d}v}}{e^{\int_0^\omega b_j(u)\,\mathrm{d}u}-1},\qquad j=1,2,\cdots,p. \tag{3.3}$$

事实上, 注意到方程 (2.1b) 是一阶线性方程且 $y_j(t)=y_j(t+\omega)$, 我们可通过解方程 (2.1b) 得到方程 (3.1). 另一方面, 我们可通过对方程 (3.1) 求导, 能获得方程 (2.1b). 因此, 系统 (2.1) 的 ω 周期解的存在性等价于下列系统 ω 周期解的存在性

$$\dot{x}_i(t)=-a_ix_i(t)+\sum_{j=1}^{p}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_i(t), \tag{3.4}$$

$i=1,2,\cdots,n$. 如果 (3.4) 有 ω 周期解 $x=\big(x_1(t),x_2(t),\cdots,x_n(t)\big)^T$, 我们可根据 (3.1) 验证 $y_j(t)$ $(j=1,2,\cdots,p.)$ 是方程 (2.1b) 的 ω 周期解. 这样 $(x_i(t)_{i=1}^n,y_j(t)_{j=1}^p)$ 是系统 (2.1) 的 ω 周期解. 所以, 我们仅需研究方程 (3.4) 的 ω 周期解存在性.

令 $x(t)=\big(x_1(t),x_2(t),\cdots,x_n(t)\big)^T$, 为应用引理 1 到系统 (2.1), 令

$$X=\big\{x(t)\in C(R,R^n):\ x_i(t+\omega)=x_i(t),\ i=1,2,\cdots,n,\ \text{对某个}\ \omega\ge 0\big\}.$$

其范数 $\|x\|=\sum\limits_{i=1}^{n}\max\limits_{t\in[0,\omega]}\big|x_i(t)\big|$, 则 X 是 Banach 空间.

令

$$N\begin{pmatrix}x_1\\x_2\\\cdots\\x_n\end{pmatrix}=\begin{pmatrix}-a_1x_1(t)+\sum\limits_{j=1}^{p}p_{j1}(t)f_j\Big(\int_{t-\tau_{j1}(t)}^{t-\tau_{j1}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_1(t)\\-a_2x_2(t)+\sum\limits_{j=1}^{p}p_{j2}(t)f_j\Big(\int_{t-\tau_{j2}(t)}^{t-\tau_{j2}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_2(t)\\\vdots\\-a_nx_n(t)+\sum\limits_{j=1}^{p}p_{jn}(t)f_j\Big(\int_{t-\tau_{jn}(t)}^{t-\tau_{jn}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_n(t)\end{pmatrix},$$

$$L\begin{pmatrix}x_1\\x_2\\\cdots\\x_n\end{pmatrix}=\begin{pmatrix}x_1'\\x_2'\\\cdots\\x_n'\end{pmatrix},\qquad P\begin{pmatrix}x_1\\x_2\\\cdots\\x_n\end{pmatrix}=Q\begin{pmatrix}x_1\\x_2\\\cdots\\x_n\end{pmatrix}=\begin{pmatrix}\frac{1}{\omega}\int_0^{\omega}x_1(t)\,\mathrm{d}t\\\frac{1}{\omega}\int_0^{\omega}x_2(t)\,\mathrm{d}t\\\cdots\\\frac{1}{\omega}\int_0^{\omega}x_n(t)\,\mathrm{d}t\end{pmatrix},\qquad x\in X.$$

容易证明 L 是指标为零的 Fredholm 算子，$P:X\cap\mathrm{Dom\,L}\to\mathrm{Ker\,L}$ 和 $Q:X\to \mathrm{X/Im\,L}$ 是两个投影，且 N 对 X 中任意给定的有界开集 Ω 在 $\overline{\Omega}$ 上是 L 紧的.

对应于方程 (2.3)，有

$$\dot{x}_i(t)=-\lambda a_ix_i(t)+\lambda\sum_{j=1}^{p}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+\lambda I_i(t).\tag{3.5}$$

假设 $x(t)\in X$ 是方程 (3.5) 关于某个 $\lambda\in(0,1)$ 的一个解，为方便起见，定义

$$\|x\|_2=\Big(\int_0^{\omega}|x(t)|^2\,\mathrm{d}t\Big)^{\frac{1}{2}},\qquad 对\ x\in C(R,R).$$

用 $\dot{x}_i$ 乘方程 (3.5) 两边并从 $[0,\omega]$ 积分，可得

$$\begin{aligned}\|\dot{x}_i\|_2^2=&\lambda\int_0^{\omega}\dot{x}_i(t)\Big[\sum_{j=1}^{p}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_i(t)\Big]\,\mathrm{d}t\\
\le&\sum_{j=1}^{p}p_{ji}^{+}\mu_j\int_0^{\omega}\Big[|\dot{x}_i(t)|\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)|\beta_j(s,x)|\,\mathrm{d}s\Big]\,\mathrm{d}t+I_i^{+}\int_0^{\omega}|\dot{x}_i(t)|\,\mathrm{d}t\\
\le&\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\int_0^{\omega}\Big[|\dot{x}_i(t)|\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}|\beta_j(s,x)|\,\mathrm{d}s\Big]\,\mathrm{d}t+I_i^{+}\sqrt{\omega}\|\dot{x}_i\|_2\\
\le&\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\int_0^{\omega}\Big[|\dot{x}_i(t)|\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}\Big(\sum_{l=1}^{n}q_{lj}^{+}\mu_l|x_l(s-\sigma_{lj}(s))|+J_j^{+}\Big)\,\mathrm{d}s\Big]\,\mathrm{d}t\\
&+I_i^{+}\sqrt{\omega}\|\dot{x}_i\|_2\\
=&\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\int_0^{\omega}|\dot{x}_i(t)|\,\mathrm{d}t\int_0^{\omega}\Big(\sum_{l=1}^{n}q_{lj}^{+}\mu_l|x_l(s-\sigma_{lj}(s))|+J_j^{+}\Big)\,\mathrm{d}s+I_i^{+}\sqrt{\omega}\|\dot{x}_i\|_2.\end{aligned}$$

利用 Cauchy 不等式可得

$$\begin{aligned}\|\dot{x}_i\|_2^2 \le& \sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\sqrt{\omega}\Big[\sum_{l=1}^{n} q_{lj}^{+}\mu_l\sqrt{\omega}\Big(\int_0^{\omega} x_l^2\big(s-\sigma_{lj}(s)\big)\,\mathrm{d}s\Big)^{1/2}\Big]\|\dot{x}_i\|_2\\ &+\Big(\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\omega\sqrt{\omega}J_j^{+}+\sqrt{\omega}I_i^{+})\|\dot{x}_i\|_2\\ =&\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\sqrt{\omega}\Big[\sum_{l=1}^{n} q_{lj}^{+}\mu_l\sqrt{\omega}\Big(\int_0^{\omega} x_l^2\big(s-\sigma_{lj}(s)\big)\frac{1}{1-\dot{\sigma}_{lj}(s)}\,\mathrm{d}\big(s-\sigma_{lj}(s)\big)\Big)^{1/2}\Big]\|\dot{x}_i\|_2\\ &+\Big(\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\omega\sqrt{\omega}J_j^{+}+\sqrt{\omega}I_i^{+}\Big)\|\dot{x}_i\|_2.\end{aligned}$$

注意到

$$k_{ij}=\Big(\max_{0\le t\le\omega}\frac{1}{1-\dot{\sigma}_{ij}(t)}\Big)^{\frac{1}{2}},\qquad \dot{\sigma}_{lj}(s)<1,$$

则有

$$\begin{aligned}\|\dot{x}_i\|_2^2 \le& \sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\sqrt{\omega}\Big[\sum_{l=1}^{n} q_{lj}^{+}\mu_l\sqrt{\omega}k_{lj}\Big(\int_{-\sigma_{lj}(0)}^{\omega-\sigma_{lj}(\omega)} x_l^2(u)\,\mathrm{d}u\Big)^{1/2}\Big]\|\dot{x}_i\|_2\\ &+\Big(\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j\omega\sqrt{\omega}J_j^{+}+\sqrt{\omega}I_i^{+})\|\dot{x}_i\|_2\\ =&\sum_{j=1}^{p}\sum_{l=1}^{n} k_{lj}p_{ji}^{+}\mu_j\alpha_j q_{lj}^{+}\mu_l\omega\|x_l\|_2\|\dot{x}_i\|_2+\Big(\omega\sqrt{\omega}\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j J_j^{+}+\sqrt{\omega}I_i^{+}\Big)\|\dot{x}_i\|_2.\end{aligned}$$

由此得到

$$\begin{aligned}\|\dot{x}_i\|_2 \le& \sum_{j=1}^{p}\sum_{l=1}^{n} k_{lj}p_{ji}^{+}\mu_j\alpha_j q_{lj}^{+}\mu_l\omega\|x_l\|_2+\omega\sqrt{\omega}\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_{ji}J_j^{+}+\sqrt{\omega}I_i^{+}\\ =&\sum_{l=1}^{n}\Big(\omega\mu_l\sum_{j=1}^{p} k_{lj}p_{ji}^{+}\mu_j\alpha_j q_{lj}^{+}\Big)\|x_l\|_2+\omega\sqrt{\omega}\sum_{j=1}^{p} p_{ji}^{+}\alpha_j\mu_j J_j^{+}+\sqrt{\omega}I_i^{+}\\ =&\sum_{l=1}^{n} c_{il}^{*}\|x_l\|_2+d_i^{*},\end{aligned}\tag{3.6}$$

其中

$$c_{il}^{*}=\omega\mu_l\sum_{j=1}^{p} k_{lj}p_{ji}^{+}\mu_j\alpha_j q_{lj}^{+},\qquad d_i^{*}=\omega\sqrt{\omega}\sum_{j=1}^{p} p_{ji}^{+}\mu_j\alpha_j J_j^{+}+\sqrt{\omega}I_i^{+}.$$

从 0 到 ω 积分 (3.5) 式有

$$a_i\int_0^{\omega} x_i(t)\,\mathrm{d}t=\int_0^{\omega}\Big[\sum_{j=1}^{p} p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega} K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_i(t)\Big]\,\mathrm{d}t.$$

这样存在 $\xi\in(0,\omega)$ 使得

$$a_i\omega x_i(\xi)=\int_0^{\omega}\Big[\sum_{j=1}^{p}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)+I_i(t)\Big]\,\mathrm{d}t.$$

因此,

$$\begin{aligned}\omega a_i\big|x_i(\xi)\big|&\le\sum_{j=1}^{p}p_{ji}^{+}\mu_j\int_0^{\omega}\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\,\mathrm{d}t+I_i^{+}\omega\\&\le\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\int_0^{\omega}\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}\Big(\sum_{l=1}^{n}q_{lj}^{+}\mu_l\big|x_l\big(s-\sigma_{lj}(s)\big)\big|+J_j^{+}\Big)\,\mathrm{d}s\,\mathrm{d}t+\omega I_i^{+}\\&=\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\sum_{l=1}^{n}q_{lj}^{+}\mu_l\int_0^{\omega}\int_0^{\omega}\big|x_l\big(s-\sigma_{lj}(s)\big)\big|\,\mathrm{d}s\,\mathrm{d}t+\omega^2\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}+\omega I_i^{+}\\&=\omega\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\sum_{l=1}^{n}q_{lj}^{+}\mu_l\int_0^{\omega}\big|x_l\big(s-\sigma_{lj}(s)\big)\big|\,\mathrm{d}s+\omega^2\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}+\omega I_i^{+}.\end{aligned}$$

再利用 Cauchy 不等式有

$$\begin{aligned}\omega a_i\big|x_i(\xi)\big|&\le\omega\sqrt{\omega}\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\sum_{l=1}^{n}q_{lj}^{+}\mu_l\Big(\int_0^{\omega}x_l^2\big(s-\sigma_{lj}(s)\big)\,\mathrm{d}s\Big)^{\frac12}+\omega^2\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}+\omega I_i^{+}\\&\le\omega\sqrt{\omega}\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\sum_{l=1}^{n}q_{lj}^{+}\mu_lk_{lj}\Big(\int_{-\sigma_{lj}(0)}^{-\sigma_{lj}(\omega)+\omega}x_l^2(u)\,\mathrm{d}u\Big)^{\frac12}\\&\quad+\omega^2\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}+\omega I_i^{+}\\&\le\sqrt{\omega}\Big\{\sum_{l=1}^{n}\Big(\omega\mu_l\sum_{j=1}^{p}k_{lj}p_{ji}^{+}\mu_j\alpha_jq_{lj}^{+}\Big)\|x_l\|_2+\omega\sqrt{\omega}\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}+\sqrt{\omega}I_i^{+}\Big\}\\&=\sqrt{\omega}\Big\{\sum_{l=1}^{n}c_{il}^{*}\|x_l\|_2+d_i^{*}\Big\}.\end{aligned}$$

即

$$\sqrt{\omega}a_i\big|x_i(\xi)\big|\le\sum_{l=1}^{n}c_{il}^{*}\|x_l\|_2+d_i^{*}.\tag{3.7}$$

又因为对 $t\in[0,\omega]$,

$$\big|x_i(t)\big|\le\big|x_i(\xi)\big|+\int_0^{\omega}\big|\dot{x}_i(t)\big|\,\mathrm{d}t\le\big|x_i(\xi)\big|+\sqrt{\omega}\|\dot{x}_i\|_2.\tag{3.8}$$

因此,

$$a_i\|x_i\|_2\le\sqrt{\omega}a_i\max_{t\in[0,\omega]}\big|x_i(t)\big|\le\sqrt{\omega}a_i\big|x_i(\xi)\big|+\omega a_i\|\dot{x}_i\|_2$$

$$\leq \sum_{l=1}^{n} c_{il}^{*}\|x_l\|_2 + d_i^{*} + \omega a_i\|\dot{x}_i\|_2. \tag{3.9}$$

把 (3.6) 式代入 (3.9) 式得

$$\begin{aligned} a_i\|x_i\|_2 &\leq \sum_{l=1}^{n} c_{il}^{*}\|x_l\|_2 + d_i^{*} + \omega a_i\Big(\sum_{l=1}^{n} c_{il}^{*}\|x_l\|_2 + d_i^{*}\Big) \\ &= \sum_{l=1}^{n}(1+\omega a_i)c_{il}^{*}\|x_l\|_2 + (1+\omega a_i)d_i^{*}. \end{aligned}$$

从而,

$$\sum_{l=1}^{n}\left[a_i\delta_{il} - (1+\omega a_i)c_{il}^{*}\right]\|x_l\|_2 \leq (1+\omega a_i)\,d_i^{*} = d_i,$$

即

$$\sum_{l=1}^{n} c_{il}\|x_l\|_2 \leq d_i, \qquad i = 1,2,\cdots,n, \tag{3.10}$$

其中

$$c_{il} = a_i\delta_{il} - (1+\omega a_i)c_{il}^{*}, \qquad \delta_{il} = \begin{cases} 1, & i = l, \\ 0, & i \neq l. \end{cases}$$

把 (3.10) 写成矩阵形式

$$CZ \leq d, \tag{3.11}$$

其中 $C = (c_{il})_{n\times n}$, $d = (d_1, d_2, \cdots, d_n)^T$, $Z = (\|x_1\|_2, \|x_2\|_2, \cdots, \|x_n\|_2)^T$, 从条件 (i) 及引理 2 有

$$Z \leq C^{-1}d = (R_1^{+}, R_2^{+}, \cdots, R_n^{+})^T,$$

即

$$\|x_i\|_2 \leq R_i^{+}, \qquad i = 1,2,\cdots,n. \tag{3.12}$$

从 (3.6)-(3.8) 和 (3.12) 知存在正常数 R_i $(i = 1,2,\cdots,n.)$ 满足

$$|x_i(t)| \leq R_i, \qquad i = 1,2,\cdots,n. \tag{3.13}$$

显然, R_i $(i = 1,2,\cdots,n)$ 不依赖于参数 λ. 令 $M^{*} = \sum_{i=1}^{n} R_i + R_0$, 这里 $R_0 > 0$ 取得充分大使得

$$\min_{1\leq i\leq n}\Big\{a_i - \omega\sum_{j=1}^{p}\sum_{l=1}^{n} k_{lj}p_{jl}^{+}\mu_j\alpha_j q_{ij}^{+}\mu_i\Big\}M^{*} > \sum_{i=1}^{n}\Big(|\widetilde{I}_i| + \sum_{j=1}^{p}|p_{ji}^{+}\mu_j J_j^{+}\alpha_j\Big)\omega. \tag{3.14}$$

取 $\Omega = \{(x(t) \in X : \|x\| < M^{*}\}$, 这样引理 1 的条件 (a) 被满足. 另一方面, 当

$x\in\partial\Omega\cap\operatorname{Ker}L=\partial\Omega\cap R^n$ 时， x 是 R^n 中的常向量且 $\sum\limits_{i=1}^{n}|x_i|=M^*$, 从而

$$\begin{aligned}QNx=&\Big(\frac{1}{\omega}\int_0^{\omega}\Big[-a_ix_i+\sum_{j=1}^{p}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s_j\Big)+I_i(t)\Big]\,\mathrm{d}t\Big)_{n\times1}\\=&\Big(-a_ix_i+\sum_{j=1}^{p}\frac{1}{\omega}\int_0^{\omega}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)\,\mathrm{d}t+\widetilde{I}_i\Big)_{n\times1}.\end{aligned}$$

因此,

$$\begin{aligned}&\|QNx\|\\=&\sum_{i=1}^{n}\Big|-a_ix_i+\sum_{j=1}^{p}\frac{1}{\omega}\int_0^{\omega}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)\,\mathrm{d}t+\widetilde{I}_i\Big|\\\geq&\sum_{i=1}^{n}\Big[a_i|x_i|-\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\frac{1}{\omega}\int_0^{\omega}\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}|\beta_j(s,x)|\,\mathrm{d}s\,\mathrm{d}t-|\widetilde{I}_i|\Big]\\\geq&\sum_{i=1}^{n}\Big\{a_i|x_i|-\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\Big[\frac{1}{\omega}\int_0^{\omega}\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}\sum_{l=1}^{n}q_{lj}^{+}\mu_l\big|x_l\big(s-\sigma_{lj}(s)\big)\big|\,\mathrm{d}s\,\mathrm{d}t+J_j^{+}\omega\Big]-|\widetilde{I}_i|\Big\}\\=&\sum_{i=1}^{n}\Big\{\Big[a_i|x_i|-\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_j\Big[\frac{1}{\omega}\int_0^{\omega}\int_0^{\omega}\sum_{l=1}^{n}q_{lj}^{+}\mu_l\big|x_l\big(s-\sigma_{lj}(s)\big)\big|\,\mathrm{d}s\,\mathrm{d}t+J_j^{+}\omega\Big]-|\widetilde{I}_i|\Big\}\\=&\sum_{i=1}^{n}a_i|x_i|-\sum_{i=1}^{n}\sum_{j=1}^{p}\sum_{l=1}^{n}p_{ji}^{+}\mu_j\alpha_jq_{lj}^{+}\mu_l\int_0^{\omega}\big|x_l\big(s-\sigma_{lj}(s)\big)\big|\,\mathrm{d}s-\sum_{i=1}^{n}\Big(|\widetilde{I}_i|+\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}\omega\Big)\\=&\sum_{i=1}^{n}a_i|x_i|-\omega\sum_{i=1}^{n}\sum_{j=1}^{p}\sum_{l=1}^{n}p_{ji}^{+}\mu_j\alpha_jq_{lj}^{+}\mu_l|x_l|-\sum_{i=1}^{n}\Big(|\widetilde{I}_i|+\sum_{j=1}^{p}p_{ji}^{+}\mu_j\alpha_jJ_j^{+}\omega\Big)\\\geq&\min_{1\leq i\leq n}\Big\{a_i-\omega\sum_{j=1}^{p}\sum_{l=1}^{n}p_{jl}^{+}\mu_j\alpha_jq_{ij}^{+}\mu_i\Big\}\sum_{i=1}^{n}|x_i|-\sum_{i=1}^{n}\Big(|\widetilde{I}_i|+\sum_{j=1}^{p}p_{ji}^{+}\mu_jJ_j^{+}\alpha_j\omega\Big)\\>&0.\end{aligned}$$

故有， $QNx\neq0$, 对 $x\in\partial\Omega\cap\operatorname{Ker}\mathrm{L}$. 这样引理 1 的条件 (b) 被满足.

定义映射 $\Phi:\operatorname{Dom}\mathrm{L}\times[0,1]\to X$,

$$\begin{aligned}\Phi(x,\mu)=&\mu(-a_ix_i)_{n\times1}\\&+(1-\mu)\Big(-a_ix_i+\sum_{j=1}^{p}\frac{1}{\omega}\int_0^{\omega}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)\,\mathrm{d}t+\widetilde{I}_i\Big)_{n\times1},\end{aligned}$$

其中 $x\in R^n$, 当 $x\in\partial\Omega\cap\operatorname{Ker}\mathrm{L}$ 且 $\mu\in[0,1]$ 时， x 是 R^n 中的常值向量且 $\sum\limits_{i=1}^{n}|x_i|=M^*$.

因此,

$$\big\|\Phi(x,\mu)\big\|=\sum_{i=1}^{n}\Big|-a_i\mu x_i+(1-\mu)$$

$$\cdot\Big(-a_ix_i+\sum_{j=1}^{p}\frac{1}{\omega}\int_0^{\omega}p_{ji}(t)f_j\Big(\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\beta_j(s,x)\,\mathrm{d}s\Big)\,\mathrm{d}t+\widetilde{I}_i\Big)\Big|$$

$$\geq\sum_{i=1}^{n}\Big\{-a_i|x_i|-\sum_{j=1}^{p}\frac{1}{\omega}\int_0^{\omega}p_{ji}^{+}\mu_j\int_{t-\tau_{ji}(t)}^{t-\tau_{ji}(t)+\omega}K_j(t,s)\big|\beta_j(s,x)\big|\,\mathrm{d}s\,\mathrm{d}t-|\widetilde{I}_i|\Big\}$$

$$\geq\min_{1\leq i\leq n}\Big\{a_i-\omega\sum_{j=1}^{p}\sum_{l=1}^{n}p_{jl}^{+}\mu_j\alpha_jq_{ij}^{+}\mu_i\Big\}\sum_{i=1}^{n}|x_i|-\sum_{i=1}^{n}\Big(|\widetilde{I}_i|+\sum_{j=1}^{p}p_{ji}^{+}\mu_jJ_j^{+}\alpha_j\omega\Big)$$

$$>0.$$

这样，我们有 $\deg\big(QNx,\Omega\cap\mathrm{Ker}\,L,0\big)=\deg\big((-a_1x_1,\cdots,-a_nx_n)^T,\Omega\cap KerL,0\big)\neq 0$. 至此，引理 1 的所有条件满足. 因此，系统 (2.1), (2.2) 至少存在一个 ω 周期解.

4 周期解的全局指数稳定性

定理 2 假设定理 1 的条件满足,若存在常数 λ_i 与 λ_{n+j} $(i=1,2,\cdots,n;\ j=1,2,\cdots,p)$ 满足:

$$\lambda_i\Big(-ra_i+(r-1)\sum_{j=1}^{p}p_{ji}^{+}\mu_j\Big)+\sum_{j=1}^{p}\lambda_{n+j}\frac{q_{ij}^{+}\mu_i}{1-\dot{\sigma}_{ij}^{+}}<0, \tag{4.1a}$$

$$\lambda_{n+j}\Big(-rb_j^{-}+(r-1)\sum_{i=1}^{n}q_{ij}^{+}\mu_i\Big)+\sum_{i=1}^{n}\lambda_i\frac{p_{ji}^{+}\mu_j}{1-\dot{\tau}_{ji}^{+}}<0, \tag{4.1b}$$

其中 r 为大于 1 的常数，则系统 (2.1) 的 ω 周期解是全局指数稳定的.

证 设 $z^*=(x_1^*(t),\cdots,x_n^*(t),y_1^*(t),\cdots,y_p^*(t))^T$ 是系统 (2.1) 的 ω 周期解, $z=(x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_p(t))^T$ 是系统 (2.1) 的任一解. 令

$$u_i(t)=x_i(t)-x_i^*(t),\qquad v_j(t)=y_j(t)-y_j^*(t),$$
$$g_i\big(u_i(t)\big)=f_i\big(u_i(t)+x_i^*(t)\big)-f_i\big(x_i^*(t)\big),\qquad g_j\big(v_j(t)\big)=f_j\big(v_j(t)+y_j^*(t)\big)-f_j\big(y_j^*(t)\big),$$

对 $i=1,2,\cdots,n;\ j=1,2,\cdots,p$. 由系统 (2.1) 可得

$$\dot{u}_i(t)=-a_iu_i(t)+\sum_{j=1}^{p}p_{ji}(t)g_j\big(v_j(t-\tau_{ji}(t))\big), \tag{4.2a}$$

$$\dot{v}_j(t)=-b_j(t)v_j(t)+\sum_{i=1}^{n}q_{ij}(t)g_i\big(u_i(t-\sigma_{ij}(t))\big). \tag{4.2b}$$

由 (H1), 有

$$\big|g_i\big(u_i(t-\sigma_{ij})\big)\big|\leq\mu_i\big|u_i(t-\sigma_{ij})\big|,\qquad \big|g_j\big(v_j(t-\tau_{ij})\big)\big|\leq\mu_j\big|v_j(t-\tau_{ji})\big|,$$

$i=1,2,\cdots,n;\ j=1,2,\cdots,p$. 注意到 (4.1), 我们可选择适当少的常数 $\varepsilon>0$ 使得

$$\lambda_i\Big(\varepsilon-ra_i+(r-1)\sum_{j=1}^{p}p_{ji}^{+}\mu_j\Big)+e^{\varepsilon\sigma}\sum_{j=1}^{p}\lambda_{n+j}\frac{q_{ij}^{+}\mu_i}{1-\dot{\sigma}_{ij}^{+}}<0, \tag{4.3a}$$

$$\lambda_{n+j}\Big(\varepsilon - r b_j^- + (r-1)\sum_{i=1}^{n} q_{ij}^+\mu_i\Big) + e^{\varepsilon\tau}\sum_{i=1}^{n}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+} < 0. \tag{4.3b}$$

构造李亚普洛夫函数 $V(t)$,

$$\begin{aligned} &V(t) = V_1(t) + V_2(t), \\ &V_1(t) = V_{11}(t) + V_{12}(t), \qquad V_2(t) = V_{21}(t) + V_{22}(t), \end{aligned} \tag{4.4}$$

其中

$$V_{11}(t) = \sum_{i=1}^{n}\lambda_i\big|u_i(t)\big|^r e^{\varepsilon t}, \qquad V_{12}(t) = \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}\int_{t-\tau_{ji}(t)}^{t}\big|v_j(s)\big|^r e^{\varepsilon(s+\tau_{ji}^+)}\,\mathrm{d}s,$$

$$V_{21}(t) = \sum_{j=1}^{p}\lambda_{n+j}\big|v_j(t)\big|^r e^{\varepsilon t}, \qquad V_{22}(t) = \sum_{j=1}^{p}\sum_{i=1}^{n}\lambda_{n+j}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ij}^+}\int_{t-\sigma_{ij}(t)}^{t}\big|u_i(s)\big|^r e^{\varepsilon(s+\sigma_{ij}^+)}\,\mathrm{d}s,$$

计算 $V_{ij}(t)$ $(i=1,2)$. 沿 (4. 2) 的解的 Dini 导数，可得

$$\begin{aligned} &D^+V_{11}(t)|_{(4.2)} \\ \le &\sum_{i=1}^{n}\lambda_i r\big|u_i(t)\big|^{r-1}e^{\varepsilon t}D^+\big|u_i(t)\big| + \varepsilon e^{\varepsilon t}\sum_{i=1}^{n}\lambda_i\big|u_i(t)\big|^r \\ \le &e^{\varepsilon t}\Big\{\sum_{i=1}^{n}\lambda_i r\big|u_i(t)\big|^{r-1}\operatorname{sign}u_i\Big[-a_iu_i(t) + \sum_{j=1}^{p}p_{ji}(t)g_j\big(v_j\big(t-\tau_{ji}(t)\big)\big)\Big] + \varepsilon\sum_{i=1}^{n}\lambda_i\big|u_i(t)\big|^r\Big\} \\ \le &e^{\varepsilon t}\Big\{\sum_{i=1}^{n}\lambda_i(\varepsilon - ra_i)\big|u_i(t)\big|^r + \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i p_{ji}^+\mu_j r\big|u_i(t)\big|^{r-1}\big|v_j\big(t-\tau_{ji}(t)\big)\big|\Big\}. \end{aligned}$$

利用 Yang 不等式 $ab \le \frac{1}{r}a^r + \frac{r-1}{r}b^{\frac{r}{r-1}}$, $a>0$, $b>0$, $r>1$. 我们有

$$\begin{aligned} &D^+V_{11}(t)|_{(4.2)} \\ \le &e^{\varepsilon t}\Big\{\sum_{i=1}^{n}\lambda_i(\varepsilon - ra_i)\big|u_i(t)\big|^r + \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i p_{ji}^+\mu_j\big[(r-1)\big|u_i(t)\big|^r + \big|v_j\big(t-\tau_{ji}(t)\big)\big|^r\big]\Big\} \\ = &e^{\varepsilon t}\sum_{i=1}^{n}\lambda_i\Big(\varepsilon - ra_i + (r-1)\sum_{j=1}^{p}p_{ji}^+\mu_j\Big)\big|u_i(t)\big|^r + e^{\varepsilon t}\sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i p_{ji}^+\mu_j\big|v_j\big(t-\tau_{ji}(t)\big)\big|^r. \end{aligned} \tag{4.5}$$

再注意到 $1-\dot{\tau}_{ji}(t) \ge 1-\dot{\tau}_{ji}^+$ 与 $e^{\varepsilon(t-\tau_{ji}(t)+\tau_{ji}^+)} \ge e^{\varepsilon t}$, 则有

$$\begin{aligned} &D^+V_{12}(t)|_{(4.2)} \\ \le &\sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}\big|v_j(t)\big|^r e^{\varepsilon(t+\tau_{ji}^+)} - \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i p_{ji}^+\mu_j\frac{1-\dot{\tau}_{ji}(t)}{1-\dot{\tau}_{ji}^+}\big|v_j(t-\tau_{ji}(t))\big|^r e^{\varepsilon t} \\ \le &e^{\varepsilon t}\sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}e^{\varepsilon\tau}\big|v_j(t)\big|^r - e^{\varepsilon t}\sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i p_{ji}^+\mu_j\big|v_j\big(t-\tau_{ji}(t)\big)\big|^r. \end{aligned} \tag{4.6}$$

从 (4.5), (4.6) 可得

$$D^+V_1(t)\big|_{(4.2)} \le e^{\varepsilon t}\sum_{i=1}^{n}\lambda_i\Big(\varepsilon - ra_i + (r-1)\sum_{j=1}^{p}p_{ji}^+\mu_j\Big)\big|u_i(t)\big|^r + e^{\varepsilon t}\sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}e^{\varepsilon\tau}\big|v_j(t)\big|^r. \tag{4.7}$$

类似地,

$$D^+V_2(t)\big|_{(4.2)} \le e^{\varepsilon t}\sum_{j=1}^{p}\lambda_{n+j}\Big(\varepsilon - rb_j^- + (r-1)\sum_{i=1}^{n}q_{ij}^+\mu_i\Big)\big|v_j(t)\big|^r + e^{\varepsilon t}\sum_{j=1}^{p}\sum_{i=1}^{n}\lambda_{n+j}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ji}^+}e^{\varepsilon\sigma}|u_i(t)|^r. \tag{4.8}$$

从 (4.7)-(4.8) 式得到

$$\begin{aligned} D^+V(t)|_{(4.2)} \le& D^+V_1(t)|_{(4.2)} + D^+V_2(t)|_{(4.2)} \\ \le& e^{\varepsilon t}\sum_{i=1}^{n}\Big[\lambda_i\Big(\varepsilon - ra_i + (r-1)\sum_{j=1}^{p}p_{ji}^+\mu_j\Big) + e^{\varepsilon\sigma}\sum_{j=1}^{p}\lambda_{n+j}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ij}^+}\Big]\big|u_i(t)\big|^r \\ &+ e^{\varepsilon t}\sum_{j=1}^{p}\Big[\lambda_{n+j}\Big(\varepsilon - rb_j^- + (r-1)\sum_{i=1}^{n}q_{ij}^+\mu_i\Big) + e^{\varepsilon\tau}\sum_{i=1}^{n}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}\Big]\big|u_i(t)\big|^r \\ <& 0. \end{aligned} \tag{4.9}$$

即, $V(t)\le V(0)$ 对 $t\ge 0$. 显然,

$$V(t) \ge \min_{1\le k\le n+p}\{\lambda_k\}e^{\varepsilon t}\Big\{\sum_{i=1}^{n}\big|u_i(t)\big|^r + \sum_{j=1}^{p}\big|v_j(t)\big|^r\Big\}. \tag{4.10}$$

另一方面, 从 (4.4) 式可得

$$\begin{aligned} V(0) =& \sum_{i=1}^{n}\lambda_i\big|u_i(0)\big|^r + \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}\int_{-\tau_{ji}(0)}^{0}\big|v_j(s)\big|^r e^{\varepsilon(s+\tau_{ji}^+)}\,\mathrm{d}s \\ &+ \sum_{j=1}^{p}\lambda_{n+j}\big|v_j(0)\big|^r + \sum_{j=1}^{p}\sum_{i=1}^{n}\lambda_{n+j}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ij}^+}\int_{-\sigma_{ij}(0)}^{0}\big|u_i(s)\big|^r e^{\varepsilon(s+\sigma_{ij}^+)}\,\mathrm{d}s \\ \le& \sum_{i=1}^{n}\lambda_i\big|u_i(0)\big|^r + \sum_{i=1}^{n}\sum_{j=1}^{p}\lambda_i\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}e^{\varepsilon\tau}\int_{-\tau}^{0}\big|v_j(s)\big|^r e^{\varepsilon s}\,\mathrm{d}s \\ &+ \sum_{j=1}^{p}\lambda_{n+j}\big|v_j(0)\big|^r + \sum_{j=1}^{p}\sum_{i=1}^{n}\lambda_{n+j}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ij}^+}e^{\varepsilon\sigma}\int_{-\sigma}^{0}\big|u_i(s)\big|^r e^{\varepsilon s}\,\mathrm{d}s \\ \le& \max_{1\le k\le n+p}\{\lambda_k\}\Big\{1 + \sum_{i=1}^{n}\sum_{j=1}^{p}\frac{p_{ji}^+\mu_j}{1-\dot{\tau}_{ji}^+}\tau e^{\varepsilon\tau} + \sum_{j=1}^{p}\sum_{i=1}^{n}\frac{q_{ij}^+\mu_i}{1-\dot{\sigma}_{ij}^+}\sigma e^{\varepsilon\sigma}\Big\} \end{aligned}$$

$$\times \left\|(\phi,\psi)^T-(x^*,y^*)^T\right\|_r, \tag{4.11}$$

结合 (4.10), (4.11) 有

$$\sum_{i=1}^{n}\left|u_i(t)\right|^r+\sum_{j=1}^{p}\left|v_j(t)\right|^r\le M\left\|(\phi^T,\psi^T)^T-(x^{*T},y^{*T})^T\right\|_r e^{-\varepsilon t}, \tag{4.12}$$

其中

$$M=\frac{\max\limits_{1\le k\le n+p}\{\lambda_k\}}{\min\limits_{1\le k\le n+p}\{\lambda_k\}}\Big(1+\sum_{i=1}^{n}\sum_{j=1}^{p}\frac{p_{ji}^+\mu_j}{1-\dot\tau_{ji}^+}\tau e^{\varepsilon\tau}+\sum_{j=1}^{p}\sum_{i=1}^{n}\frac{q_{ij}^+\mu_i}{1-\dot\sigma_{ij}^+}\sigma e^{\varepsilon\sigma}\Big)>1.$$

5　数值例子

考虑下列时滞 BAM 神经网络系统:

$$\dot x_i(t)=-a_ix_i(t)+\sum_{j=1}^{2}p_{ji}(t)f_j(y_j(t-\tau_{ji}(t)))+I_i(t),\qquad i=1,2. \tag{5.1a}$$

$$\dot y_j(t)=-b_j(t)y_j(t)+\sum_{i=1}^{2}q_{ij}(t)f_i\big(x_i\big(t-\sigma_{ij}(t)\big)\big)+J_j(t),\qquad j=1,2, \tag{5.1b}$$

其中, $I_i(t)=\sin t$, $J_j(t)=\cos t$, $\tau_{ji}(t)=\tau_{ji}=$ 常数, $\sigma_{ij}(t)=\sigma_{ij}=$ 常数, $a_i=1$, $b_j(t)=2+\cos t$, 则 $\dot\tau_{ji}^+=0$, $\dot\sigma_{ij}^+=0$, $k_{ij}=1$, $b_j^-=1$, $\alpha_j=\frac{e^{4\pi}}{e^{4\pi}-1}$, $(i,j=1,2)$. 令 $f_k(x)=0.4x+0.6\arctan x$, 则 $f_k(0)=0$ 且 $\mu_k=1$, $k=1,2$. 我们取

$$\begin{pmatrix}p_{11}(t)&p_{12}(t)\\p_{21}(t)&p_{22}(t)\end{pmatrix}=\begin{pmatrix}\frac{1}{10}\sin t&\frac{2}{10}\cos t\\\frac{2}{10}\cos t&\frac{1}{10}\sin t\end{pmatrix},$$

$$\begin{pmatrix}q_{11}(t)&q_{12}(t)\\q_{21}(t)&q_{22}(t)\end{pmatrix}=\begin{pmatrix}\frac{2}{10}\cos t&\frac{1}{10}\sin t\\\frac{1}{10}\sin t&\frac{2}{10}\cos t\end{pmatrix},$$

则有

$$\begin{pmatrix}p_{11}^+&p_{12}^+\\p_{21}^+&p_{22}^+\end{pmatrix}=\begin{pmatrix}\frac{1}{10}&\frac{2}{10}\\\frac{2}{10}&\frac{1}{10}\end{pmatrix},\qquad\begin{pmatrix}q_{11}^+&q_{12}^+\\q_{21}^+&q_{22}^+\end{pmatrix}=\begin{pmatrix}\frac{2}{10}&\frac{1}{10}\\\frac{1}{10}&\frac{2}{10}\end{pmatrix},$$

$$\begin{pmatrix}c_{11}^*&c_{12}^*\\c_{21}^*&c_{22}^*\end{pmatrix}=\frac{2\pi e^{4\pi}}{e^{4\pi}-1}\begin{pmatrix}\frac{1}{25}&\frac{1}{20}\\\frac{1}{20}&\frac{1}{25}\end{pmatrix},$$

$$C=\begin{pmatrix}c_{11}&c_{12}\\c_{21}&c_{22}\end{pmatrix}=\frac{2\pi e^{4\pi}}{e^{4\pi}-1}\begin{pmatrix}1-\frac{1+2\pi}{25}&-\frac{1+2\pi}{20}\\-\frac{1+2\pi}{20}&1-\frac{1+2\pi}{25}\end{pmatrix}.$$

容易计算矩阵 C 的逆矩阵,

$$C^{-1}=\frac{4200(e^{4\pi}-1)}{\pi e^{4\pi}[16(24-2\pi)^2-21(1+2\pi)^2]}\begin{pmatrix}\frac{24-2\pi}{25} & \frac{1+2\pi}{20}\\ \frac{1+2\pi}{20} & \frac{24-2\pi}{25}\end{pmatrix}>0,$$

故 C 是非奇异 M 矩阵. 另一方面, 容易验证

$$a_i-2\pi\sum_{j=1}^{p}\sum_{l=1}^{n}k_{lj}p_{jl}^{+}\mu_j\alpha_j q_{ij}^{+}\mu_i=1-\frac{18\pi e^{4\pi}}{100(e^{4\pi}-1)}>0,\qquad i=1,2.$$

由定理 1 知, 该系统至少存在一个 2π 周期解. 又取 $\lambda_i=1$, $\lambda_{2+j}=1$, $(i,j=1,2)$, $r=2$. 可得

$$\lambda_i\Big(-ra_i+(r-1)\sum_{j=1}^{p}p_{ji}^{+}\mu_j\Big)+\sum_{j=1}^{p}\lambda_{n+j}\frac{q_{ij}^{+}\mu_i}{1-\dot{\sigma}_{ij}^{+}}=-1.4<0,\qquad i=1,2,$$

$$\lambda_{n+j}\Big(-rb_j^{-}+(r-1)\sum_{i=1}^{n}q_{ij}^{+}\mu_i\Big)+\sum_{i=1}^{n}\lambda_i\frac{p_{ji}^{+}\mu_j}{1-\dot{\tau}_{ji}^{+}}=-1.4<0,\qquad j=1,2.$$

由定理 2 知该系统的 2π 周期解是全局指数稳定的.

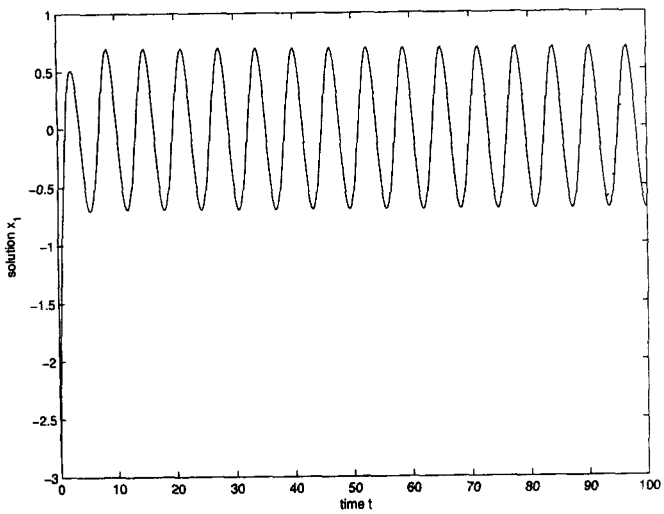

图 1 时滞 $\tau_{11}=\sigma_{11}\equiv 1$, $\tau_{21}=\sigma_{21}\equiv 2$, $\tau_{12}=\sigma_{12}\equiv 0.1$, $\tau_{22}=\sigma_{22}\equiv 0.5$, 初始值 $\phi_1(t)\equiv -3$, $\phi_2(t)\equiv -2$, $\psi_1(t)\equiv 2$, $\psi_2(t)\equiv 4$.

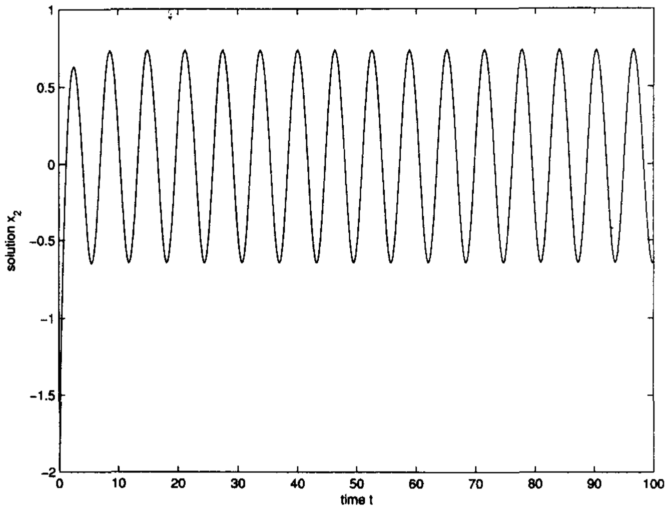

图 2 时滞 $\tau_{11}=\sigma_{11}\equiv 1$，$\tau_{21}=\sigma_{21}\equiv 2$，$\tau_{12}=\sigma_{12}\equiv 0.1$，$\tau_{22}=\sigma_{22}\equiv 0.5$，初始值 $\phi_1(t)\equiv -3$，$\phi_2(t)\equiv -2$，$\psi_1(t)\equiv 2$，$\psi_2(t)\equiv 4$.

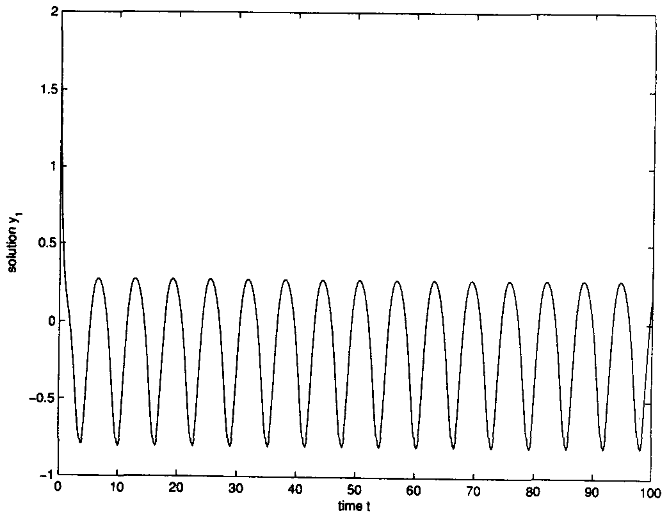

图 3 时滞 $\tau_{11}=\sigma_{11}\equiv 1$，$\tau_{21}=\sigma_{21}\equiv 2$，$\tau_{12}=\sigma_{12}\equiv 0.1$，$\tau_{22}=\sigma_{22}\equiv 0.5$，初始值 $\phi_1(t)\equiv -3$，$\phi_2(t)\equiv -2$，$\psi_1(t)\equiv 2$，$\psi_2(t)\equiv 4$.

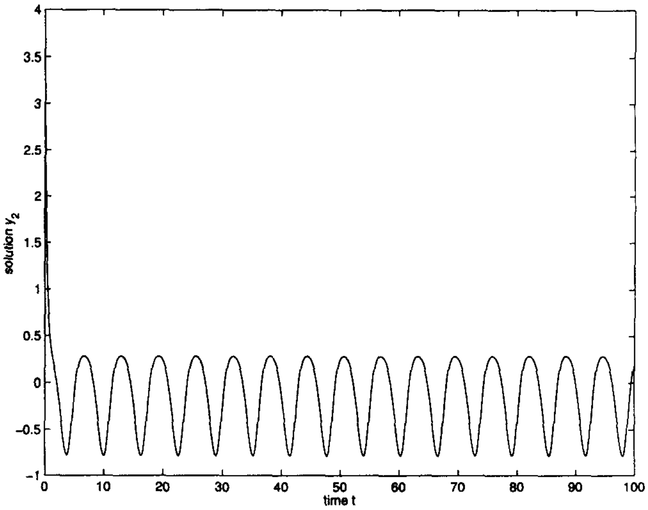

图 4 时滞 $\tau_{11}=\sigma_{11}\equiv 1$, $\tau_{21}=\sigma_{21}\equiv 2$, $\tau_{12}=\sigma_{12}\equiv 0.1$, $\tau_{22}=\sigma_{22}\equiv 0.5$, 初始值 $\phi_1(t)\equiv -3$, $\phi_2(t)\equiv -2$, $\psi_1(t)\equiv 2$, $\psi_2(t)\equiv 4$.

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EXISTENCE AND GLOBAL EXPONENTIAL STABILITY OF PERIODIC SOLUTION FOR BAM NEURAL NETWORDS WITH PERIODIC COEFFICIENTS AND DELAYS

CHEN ANPING

(*Department of Mathematics, Xiangnan University, Chenzhou* 423000)

E-mail: chenap@263.net

CAO JINDE

(*Department of Mathematics, Southeast University, Nanjing* 210096)

HUANG LIHONG

(*College of Mathematics and Econometrics, Hunan University, Changsha* 410082)

Abstract By using the continuation theorem of Mawhin's coincidence degree theory, Lyapunov functional, and combining with Yang's inequality and some analysis techniques, some sufficient conditions are obtained ensuring existence and global exponential stability of periodic solution to the BAM neural networks with periodic coefficients and delays. These results are helpful to design globally exponentially stable BAM networks and periodic oscillatory BAM neural networks.

Key words BAM neural networks; periodic solution; coincidence degree theory; global exponential stability; Lyapunov functional

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