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各向異性介質中契連科夫輻射的量子理論*

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提要

本文在各向异性介质中电磁場二次量子化的基础上，探讨做高匀速运动(超过该介质中的光速)的带电粒子辐射状况，得到了寻常波和非常波辐射强度的表达式，解释了粒子的自旋状态对辐射强度的影响.

在粒子的运动方向沿晶体光轴和垂直于晶体光轴的两种情况下，对结果进行了分析. 结果表明，量子方法的计算有别于经典结果，当考虑到粒子自旋时，在沿光轴运动的情况下，既辐射寻常波又辐射非常波.

一、引言

在一系列工作[1—4]中，应用經典方法討論了带电粒子在各向异性介質中的契連科夫輻射. 文献[1]应用哈密頓法，解各向异性介質中的麦氏方程組得到了輻射能的頻譜分布和相应的輻射条件; 文献[3,4]进一步計算了粒子通过該类介質的全部能損耗，并从中分出契連科夫輻射部分. 經典結果表明: 当粒子沿光軸运动时，仅仅存在非常波輻射，其輻射方向和強度的分布都有别于各向同性介質中的情况.

尽管經典理論已經很好地解释了該类型輻射，获得了有关輻射的基本特点，但从以下可以看出，借助于量子理論計算(相对性的或非相对性的量子电动力学)，可以得到一系列和"量子效应"相联系的輻射特点，給出新的量子补充值(即有数量級 $\hbar$, 也有数量級 $\hbar^2$).

这种討論的必要性不仅在于該物理現象本身是微观过程，量子方法很自然地計入了粒子的自旋状态、輻射光子的极化等，而且和近年来已能观测单个粒子的輻射[5]，并利用該效应找寻新的粒子，进而研究其性質相联系着. 在弱相互作用中已发現了完全縱向极化的粒子[6]，輻射能的量子补充将取决于粒子的自旋状态.

二、各向异性介質中契連科夫輻射的量子理論

設自由运动的粒子冲量为 $\mathbf{k}$, 能量 $K = \sqrt{k^2 + k_0^2}$, 輻射光子后，它处于新的量子状态 $\mathbf{k}'$, $K' = \sqrt{k'^2 + k_0^2}$ (暂不考虑自旋状态). 介質中的輻射只有在不违背如下的动量、能量守恆条件时才可能进行[7]:

$$\mathbf{k}' = \mathbf{k} - \boldsymbol{\kappa}, \quad K' = K - L, \tag{2.1}$$

式中 $\boldsymbol{\kappa} = \frac{\omega}{c}\mathbf{n}$, $L = \frac{\omega}{c}$ 分别代表輻射出的光子冲量和能量，和各向同性介質相仿，也称

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絕对值 $|n|$ 为介質的折射率(尽管它們是有区别的). 在所研究的各向异性介質具有軸对称性时(介电常数 $\varepsilon_x = \varepsilon_y = \varepsilon_0$, $\varepsilon_z = \varepsilon_c$),其寻常波和非常波的折射率分别表为[1]

$$n_0^2 = \varepsilon_0, \quad \frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{\varepsilon_0} + \frac{\sin^2\theta}{\varepsilon_e}. \tag{2.2}$$

应用文献[7]的方法,解联立方程(2.1),得到輻射的量子条件为

$$\cos\theta_{k,\lambda} = \frac{1}{\beta n_{k,\lambda}}\left[1 + \frac{\hbar\omega}{2mc^2}(n_{k,\lambda}^2 - 1)\right]. \tag{2.3}$$

若用 θ, φ 表 κ 的球形坐标角,则 θ 为初始冲量 $\mathbf{k}$ 和輻射波矢 κ 間的夾角.

由于介質在契連科夫輻射中起着重要作用(正是由于它的存在,輻射才是可能的),因此需首先討論介質中場的性質. 各向异性介質中的电动力学已在文献[1]中給出,为了微观地解释該类介質中的輻射,需对电磁場进行量子化.

将文献[1]的解(15)写成

$$\mathbf{A}(\mathbf{x}, t) = \frac{1}{L^{3/2}}\sum_{k,\lambda}(q_{k,\lambda}\beta_{k,\lambda}e^{-i\omega_{k,\lambda}t + i\kappa_k\cdot\mathbf{x}} + q_{k,\lambda}^+\beta_{k,\lambda}^+e^{i\omega_{k,\lambda}t - i\kappa_k\cdot\mathbf{x}}), \tag{2.4}$$

式中頻率[1]

$$\omega_{k,\lambda}^2 = \frac{\kappa_k^2c^2}{n_{k,\lambda}^2} = (\kappa_k^2\beta_{k,\lambda}^2 - (\kappa_k\cdot\beta_{k,\lambda})^2)\cdot c^2. \tag{2.5}$$

該表达式的特点在于很自然地从矢量 $\mathbf{A}(\mathbf{x}, t)$ 中分出量子化部分 $q_{k,\lambda}$, 而介質的各向异性就可用矢量 $\beta_{k,\lambda}$ 来描述,考虑到电磁場在該种介質中的极化态,可将其写为

$$\left.\begin{aligned}\beta_{k,0} &= \sum_{\alpha=1}^{3}\frac{n_0^2}{\varepsilon_\alpha}\frac{[\kappa^0\mathbf{j}^0]_\alpha}{\sqrt{1-(\kappa^0\mathbf{j}^0)^2}}\mathbf{e}_\alpha,\\ \beta_{k,e} &= \sum_{\alpha=1}^{3}\frac{n_e^2}{\varepsilon_\alpha}\frac{\kappa_\alpha^0(\kappa^0\mathbf{j}^0)-j_\alpha^0}{\sqrt{1-(\kappa^0\mathbf{j}^0)^2}}\mathbf{e}_\alpha,\end{aligned}\right\} \tag{2.6}$$

其中 $\mathbf{e}_\alpha = \mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ 为沿坐标軸的单位矢量; $\mathbf{j}^0$ 为沿某个选择方向的单位矢量,在我們的情况下, $\mathbf{j}^0/\!/\mathbf{k}$; $\lambda = 0, e$ 分别代表寻常波和非常波值.

这样,对电磁場进行量子化时采用已知的真空中的方法即可[7].

横向場的輻射能为[1]

$$\mathscr{H}^{tr} = \frac{1}{8\pi c^2}\int\sum_{\alpha=1}^{3}\varepsilon_\alpha\left(\frac{\partial\mathbf{A}\alpha}{\partial t}\right)\left(\frac{\partial\mathbf{A}^+\alpha}{\partial t}\right)dv + \frac{1}{8\pi}\int([\nabla\cdot\mathbf{A}]\cdot[\nabla\cdot\mathbf{A}^+])dv. \tag{2.7}$$

将 $\mathbf{A}$ 的表达式(2.4)代入,重新利用文献[1]的条件(18)—(23),便得以下的哈密頓算符:

$$\hat{\mathscr{H}}^{tr} = \frac{1}{4\pi c^2}\sum_{k,\lambda}\omega_{k,\lambda}^2(\hat{q}_{k,\lambda}\hat{q}_{k,\lambda}^+ + \hat{q}_{k,\lambda}^+\hat{q}_{k,\lambda}). \tag{2.8}$$

代入运动方程 $\hbar\omega_{k,\lambda}\hat{q}_{k,\lambda}^+ = [\hat{\mathscr{H}}^{tr}\cdot\hat{q}_{k,\lambda}^+]$ 中,便可找出振幅 $q_{k,\lambda}$ 的具体表达式

$$q_{k,\lambda} = \sqrt{\frac{2\pi\hbar c^2}{\omega_{k,\lambda}}}q_{k,\lambda}^0, \quad q_{k,\lambda}^+ = \sqrt{\frac{2\pi\hbar c^2}{\omega_{k,\lambda}}}q_{k,\lambda}^{0+}. \tag{2.9}$$

将該結果写在(2.4)內,可得横向場的量子化表达式

$$\mathbf{A}(\mathbf{x}, t) = \frac{1}{L^{3/2}}\sum_{k,\lambda}\sqrt{\frac{2\pi\hbar}{\omega_{k,\lambda}}}c\cdot(\mathbf{a}_{k,\lambda}e^{-i\omega_{k,\lambda}t+i\kappa_k\cdot\mathbf{x}} + \mathbf{a}_{k,\lambda}^+e^{i\omega_{k,\lambda}t-i\kappa_k\cdot\mathbf{x}}), \tag{2.10}$$

式中 $\mathbf{a}_{k,\lambda}=\beta_{k,\lambda}q^{0}_{k,\lambda}$. 在不存在光子的初始条件下，量子化部分满足以下的关系式：

$$[\hat{q}^{0+}_{k',\lambda'},\ \hat{q}^{0}_{k,\lambda}]=0,\qquad [\hat{q}^{0}_{k,\lambda},\ \hat{q}^{0+}_{k',\lambda'}]=\delta_{kk'}\cdot\delta_{\lambda\lambda'}. \tag{2.11}$$

从場的相互作用概念出发，在各向异性介质中仍可将相互作用能 $W'=e(\boldsymbol{\alpha}\cdot\mathbf{A})$ 視为微扰項. 这样，在微扰理論波恩一次近似中可得輻射几率公式

$$W^{s,s'}_{\lambda}=\frac{4\pi^{2}ce^{2}}{L^{3}\hbar}\sum_{\mathbf{k}}\sum_{\kappa}\frac{1}{\omega_{k,\lambda}}Q^{+s,s'}_{\lambda}\cdot Q^{s,s'}_{\lambda}\delta_{\mathbf{k},\mathbf{k}'+\kappa}\cdot\delta(K'+L-K). \tag{2.12}$$

在这里，矩陣元

$$Q^{+s,s'}_{\lambda}\cdot Q^{s,s'}_{\lambda}=\frac{1}{4}\sum_{s}\sum_{s_1}b^{+}_{s}(\boldsymbol{\alpha}\,\mathbf{a}_{k,\lambda})b'_{s'}\cdot b^{+\prime}_{s'}(\boldsymbol{\alpha}\,\mathbf{a}^{+}_{k,\lambda})b_{s_1}\cdot C_{s}C^{+}_{s_1},$$

而其中的 C_s 为系数；$\boldsymbol{\alpha}$ 为狄拉克矩陣，b_s 为自旋矩陣，它們均在文献[8]中給出. 很显然，只要重复文献[8]的运算过程，便可得到在单位长度上的輻射強度表达式

$$\begin{aligned}W^{s,s'}_{\lambda}=\frac{e^{2}}{4c^{2}}\int f(\theta)\Big\{\frac{1}{2}\sum_{s}\sum_{s_1}(1+ss_1)\Big[&A^{s,s_1}_{\lambda,n}+B^{s,s_1}_{\lambda,n}ss'\frac{1}{\Gamma}\frac{\omega n\hbar}{cp}\sin\theta+\\&+C^{s,s_1}_{\lambda,n}ss'\frac{1}{\Gamma}\Big(1-\frac{\omega n\hbar}{cp}\cos\theta\Big)\Big]\cdot\Big(1+ss'\Gamma-\frac{\hbar\omega}{cp\beta}\Big)+\\&+(1-ss_1)\Big[D^{s,s_1}_{\lambda,n}-B^{s,s_1}_{\lambda,n}s'\frac{1}{\Gamma}\Big(1-\frac{\omega n\hbar}{cp}\cos\theta\Big)+\\&+C^{s,s_1}_{\lambda,n}s'\frac{1}{\Gamma}\frac{\omega n\hbar}{cp}\sin\theta\Big]\frac{\hbar\omega}{cp\beta}\sqrt{1-\beta^{2}}\Big\}\omega d\omega,\end{aligned} \tag{2.13}$$

式中 $f(\theta)=\dfrac{1}{1-\dfrac{1}{n_{k,\lambda}(\theta)}\dfrac{\partial n_{k,\lambda}(\theta)}{\partial\theta}\operatorname{ctg}\theta}$ 乃是在計入介質的各向异性时的一个乘数；系数 $A^{s,s_1}_{\lambda,n}$, $B^{s,s_1}_{\lambda,n}$, $C^{s,s_1}_{\lambda,n}$, $D^{s,s_1}_{\lambda,n}$ 已在文献[8]的(2.9)中給出，仅是将其中的振幅分量 a_{j,n_j} 換成各向异性介質中 a_{λ,n_j} 的表达式(2.6).

三、带电粒子在单軸晶体中的輻射

正如經典結果[1]所指出的，当粒子穿过透明晶体时，若其速度超过在晶体中的光波速度，則产生复杂的輻射图景，因而，引用上节的結果討論該种輻射将是很有意思的.

粒子在单軸晶体中具有实际意义的运动方向是运动沿光軸方向和垂直于光軸方向，我們将分別討論之.

1. 粒子沿光軸运动

在单軸晶体中，寻常波电場矢量垂直于主截面振动，非常波电場矢量則平行于它. 选择 $\mathbf{j}^0$ 平行于光軸. 在垂直于光軸的平面上(設光軸沿坐标系 z 軸，則該平面乃是 xy 面)，介質是各向同性的，为了簡化問題，我們将該平面轉动一个角，使 κ_k 放置在 xz 面上($\kappa_y=$ $=0$)；这样，

$$\left.\begin{aligned}\mathbf{a}_{k,0}&=\mathbf{a}_{k,n_y}\qquad(n_x=n_z=0),\\ \mathbf{a}_{k,e}&=\mathbf{a}_{k,n_x}+\mathbf{a}_{k,n_z}\quad(n_y=0).\end{aligned}\right\} \tag{3.1}$$

算出(2.6)的相应值代入(2.9)[8]中，对于縱向自旋状态的粒子[9] ($C_1=1,\ C_{-1}=0$)，我們便得系数值

$$\left.\begin{aligned}&A^{1,\frac{1}{n}}_{0,\,n}=1,\quad B^{1,\frac{1}{n}}_{0,\,n}=0,\quad C^{1,\frac{1}{n}}_{0,\,n}=-1,\quad D^{1,\frac{1}{n}}_{0,\,n}=0,\\&A^{1,\frac{1}{n}}_{e,\,n}=\frac{n_e^2}{\varepsilon_0^2}\cos^2\theta+\frac{n_e^2}{\varepsilon_e^2}\sin^2\theta,\quad B^{1,\frac{1}{n}}_{e,\,n}=2\frac{n_e^2}{\varepsilon_0\varepsilon_e}\cos\theta\sin\theta,\\&C^{1,\frac{1}{n}}_{e,\,n}=\frac{n_e^2}{\varepsilon_e^2}\sin^2\theta-\frac{n_e^2}{\varepsilon_0^2}\cos^2\theta,\quad D^{1,\frac{1}{n}}_{e,\,n}=0.\end{aligned}\right\}\tag{3.2}$$

将該值代入(2.13),得輻射強度为

$$W^{s,s'}_{||,\,\lambda=0,\,e}=W_{||,\,\lambda=0,\,e}+ss'W'_{||,\,\lambda=0,e},\tag{3.3}$$

其中

$$\left.\begin{aligned}
W_{||,\,0}&=\frac{e^2}{4c^2}\int_0^{\omega_{\max}}\frac{\omega^2\hbar^2}{2c^2p^2}(\varepsilon_0-1)\omega d\omega,\\
W_{||,\,e}&=\frac{e^2}{4c^2}\times\\
&\times\int_0^{\omega_{\max}}\left\{\frac{2-2\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\frac{\omega\hbar}{cp\beta}-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)^2\frac{\omega^2\hbar^2}{2c^2p^2}\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\sin^2\theta+\right.\\
&\left.+\frac{\varepsilon_e^2/\varepsilon_0^2}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\frac{\omega^2\hbar^2}{2c^2p^2}\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)\right\}\omega d\omega,\\
W'_{||,\,0}&=\frac{e^2}{4c^2}\int_0^{\omega_{\max}}\frac{\omega^2\hbar^2}{2c^2p^2}\frac{1}{\Gamma}\left\{\varepsilon_0+1-\frac{2}{\beta^2}-\frac{\omega\hbar}{cp\beta}(\varepsilon_0-1)\right\}\omega d\omega,\\
W'_{||,\,e}&=\frac{e^2}{4c^2}\int_0^{\omega_{\max}}\frac{\omega\hbar}{cp\beta}\times\\
&\times\frac{1}{\Gamma}\left\{\left\{\left(1+\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\left(\frac{cp\beta}{\omega\hbar}-1\right)-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)^2\left(1-\frac{\omega\hbar}{cp\beta}\right)\times\right.\right.\\
&\left.\times\left[1+\frac{\omega\hbar\beta}{2cp}\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)\right]\right\}\sin^2\theta\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta\right]^{-1}+\\
&+\frac{\varepsilon_e^2/\varepsilon_0^2}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\frac{\omega\hbar\beta}{2cp}\left[\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}+1-\frac{2}{\beta^2}-\frac{\omega\hbar}{cp\beta}\times\right.\\
&\left.\left.\times\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)\right]\right\}\omega\,d\omega.
\end{aligned}\right\}\tag{3.4}$$

上式中的积分极限由 $\cos\theta_{0,\,e}=f(\omega_{\min}=0)<1$, $\cos\theta_{0,\,e}=f(\omega_{\max})=1$ 决定;而 $\cos\theta$, $\sin\theta$ 值决定于(2.3). W 是各向异性介質中契連科夫輻射的量子表达式, W' 乃是和粒子的自旋状态改变相联系的补充項.

和經典結果[1]的区别在于,当計入粒子的自旋状态后,和“量子效应”相联系着不仅存在非常波輻射,而且存在着寻常波輻射(式中具有常数 $\hbar$). 引入介質的各向异性后,輻射

強度表示式具有一系列补充項(不仅有数量級 $\hbar^2$,亦有数量級 $\hbar$)。

討論在閾能 ($\cos\theta = 1$) 上的輻射：从条件

$$\cos\theta = \frac{1}{\beta n_{k,\lambda}} + \frac{\omega n_{k,\lambda}\hbar}{2cp}(1 - n_{k,\lambda}^{-2}) = 1$$

得

$$\frac{1}{\beta} + \frac{\omega\hbar}{2cp}(\varepsilon_0 - 1) = \sqrt{\varepsilon_0};$$

$$\frac{1}{\beta} + \frac{\omega\hbar}{2cp}\left(\frac{\varepsilon_e}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta} - 1\right) = \sqrt{\frac{\varepsilon_e}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}}.$$

代入(3.3),(3.4)中便得

$$W_0^{s,s'} = \frac{e^2}{4c^2}\int_0^{\omega_{\max}} (1 - ss')\frac{\omega^2\hbar^2}{2c^2p^2}(\varepsilon_0 - 1)\omega\, d\omega,$$

$$W_e^{s,s'} = \frac{e^2}{4c^2}\int_0^{\omega_{\max}} \frac{\varepsilon_e^2/\varepsilon_0^2}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)}(1 - ss')\frac{\omega^2\hbar^2}{2c^2p^2}\left(\frac{\varepsilon_e}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)} - 1\right)\omega\, d\omega.$$

与各向同性介質中的結果[10]相类似,对于縱向极化的粒子 ($S = 1$), 只有自旋定向发生改变时(例如反演 $S = 1 \to S' = -1$) 輻射才有可能,在这种情况下輻射能为

$$\left.\begin{aligned} W_{0,\,\|}^{s,s'} &= \frac{e^2}{c^2}\int_0^{\omega_{\max}} \frac{\omega^2\hbar^2}{4c^2p^2}(\varepsilon_0 - 1)\,\omega\, d\omega, \\ W_{\|,e}^{s,s'} &= \frac{e^2}{c^2}\int_0^{\omega_{\max}} \frac{\varepsilon_e^2/\varepsilon_0^2}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)}\frac{\omega^2\hbar^2}{4c^2p^2}\left(\frac{\varepsilon_e}{1 - \left(1 - \frac{\varepsilon_e}{\varepsilon_0}\right)} - 1\right)\omega\, d\omega. \end{aligned}\right\} \tag{3.5}$$

对于横向极化的粒子[9] $\left(C_1 = \frac{1}{\sqrt{2}},\ C_{-1} = \frac{1}{\sqrt{2}}e^{i\varphi_0}\right)$,我們計算出系数值为

$$\left.\begin{aligned} &A_{0,n}^{s,s_1} = \frac{1}{2}, \quad C_{0,n}^{s,s_1} = -\frac{1}{2}, \quad B_{0,n}^{s,s_1} = D_{0,n}^{s,s_1} = 0, \\ &A_{0,n}^{-s,s_1} = -\frac{1}{2}e^{is_1\varphi_0}, \quad C_{0,n}^{-s,s_1} = \frac{1}{2}e^{is_1\varphi_0}, \quad B_{0,n}^{-s,s_1} = D_{0,n}^{-s,s_1} = 0, \\ &A_{e,n}^{s,s_1} = \frac{n_e^4}{2}\left(\frac{\cos^2\theta}{\varepsilon_0^2} + \frac{\sin^2\theta}{\varepsilon_e^2}\right), \quad B_{e,n}^{s,s_1} = \frac{n_e^4}{\varepsilon_0\varepsilon_e}\cos\theta\sin\theta, \\ &C_{e,n}^{s,s_1} = \frac{n_e^4}{2}\left(\frac{\sin^2\theta}{\varepsilon_e^2} - \frac{\cos^2\theta}{\varepsilon_0^2}\right), \quad D_{e,n}^{s,s_1} = 0, \\ &A_{e,n}^{-s,s_1} = \frac{n_e^4}{2}\left(\frac{\cos^2\theta}{\varepsilon_0^2} + \frac{\sin^2\theta}{\varepsilon_e^2}\right)e^{is_1\varphi_0}, \\ &B_{e,n}^{-s,s_1} = \frac{n_e^4}{\varepsilon_0\varepsilon_e}\cos\theta\sin\theta e^{is_1\varphi_0}, \\ &C_{e,n}^{-s,s_1} = \frac{n_e^4}{2}\left(\frac{\sin^2\theta}{\varepsilon_e^2} - \frac{\cos^2\theta}{\varepsilon_0^2}\right)e^{is_1\varphi_0}, \quad D_{e,n}^{-s,s_1} = 0 \quad (S = S_1 = \pm 1). \end{aligned}\right\} \tag{3.6}$$

相应的单位路程上的輻射能为

$$W^{s'}_{\|,\lambda=0,e} = W_{\|,\lambda=0,e} + S'W''_{\|,\lambda=0,e}, \tag{3.7}$$

其中

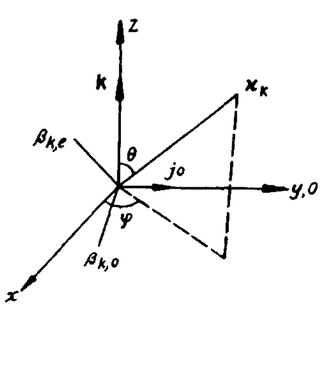
$$\left.\begin{aligned}
W''_{\|,0} &= \frac{e^2}{4c^2}\int_0^{\omega_{\max}} \frac{1}{\Gamma}\frac{\omega^2\hbar^2}{c^2p^2\beta}\sqrt{\varepsilon_0}\cdot\sqrt{1-\beta^2}\sin\theta\cos\varphi_0\cdot\omega\,d\omega,\\
W''_{\|,e} &= \frac{e^2}{4c^2}\int_0^{\omega_{\max}} \frac{1}{\Gamma}\frac{\omega\hbar}{cp\beta}\left[\frac{\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)^2\frac{\omega\hbar}{cp}}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\sqrt{\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}}\sin^2\theta-\right.\\
&\quad\left.-\frac{\varepsilon_e}{\varepsilon_0}\frac{\left(\frac{\varepsilon_e}{\varepsilon_0}-2\right)\frac{\omega\hbar}{cp}\sqrt{\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}}+2\cos\theta}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\right]\times\\
&\quad\times\sqrt{1-\beta^2}\sin\theta\cos\varphi_0\,\omega\,d\omega.
\end{aligned}\right\} \tag{3.8}$$

如果認为在輻射后粒子的自旋定向是未知的，且在实验中仅对其共同輻射強度感兴趣，則可对自旋末态求和 $\sum_{s=1}^{-1} W^{ss'}_{\lambda=0,e}$，对自旋始态取平均值，便得各向异性介质中輻射強度的量子表达式为

$$\left.\begin{aligned}
W^{s,s'}_{\|,0} &= W^{s'}_{\|,0} = \frac{e^2}{c^2}\int_0^{\omega_{\max}} \frac{\omega^3\hbar^2}{4c^2p^2}(\varepsilon_0-1)d\omega,\\
W^{s,s'}_{\|,e} &= W^{s'}_{\|,e} = \frac{e^2}{c^2}\times\\
&\times\int_0^{\omega_{\max}}\left[\frac{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\frac{\omega\hbar}{cp\beta}-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)^2\frac{\omega^2\hbar^2}{4c^2p^2}\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\sin^2\theta+\right.\\
&\left.+\frac{\varepsilon_e^2/\varepsilon_0^2}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}\frac{\omega^2\hbar^2}{4c^2p^2}\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\cos^2\theta}-1\right)\right]\omega\,d\omega.
\end{aligned}\right\} \tag{3.9}$$

将該式过渡到各向同性介质中（$\varepsilon_0=\varepsilon_e=\varepsilon$），便得到文献[8,10]的結果：

$$W^{s,s'}_{j=2,3} = W^{s'}_{j=2,3} = \frac{e^2}{c^2}\int_0^{\omega_{\max}}\left[(j-2)\sin^2\theta+\frac{\omega^2\hbar^2}{4c^2p^2}(\varepsilon-1)\right]\omega\,d\omega.$$



2. 粒子沿垂直于光軸方向运动

当粒子沿垂直于光軸方向运动时，选取 $\mathbf{j}^0$ 光軸沿 y 軸定向，而运动方向仍沿 z 軸（見左图），介质的折射率(2.2)換成

$$n_0^2 = \varepsilon_0, \quad \frac{1}{n_e^2(\theta, \varphi)} = \frac{\sin^2\theta \sin^2\varphi}{\varepsilon_0} + \frac{\sin^2\theta\cos^2\varphi + \cos^2\theta}{\varepsilon_e}, \tag{3.10}$$

和相应的 (2.6) 值一起代入系数的一般表达式 (2.9) 中[8]，对于縱向极化粒子 ($C_1 = 1$, $C_{-1} = 0$) 得

$$\left.\begin{aligned} &A_{0,\,n}^{s_1 s_1} = 1, \quad B_{0,\,n}^{s_1 s_1} = -2\frac{\sin\theta\cos\theta\cos\varphi}{1-\sin^2\theta\sin^2\varphi}, \\ &C_{0,\,n}^{s_1 s_1} = -1 + 2\frac{\sin^2\theta\cos^2\varphi}{1-\sin^2\theta\sin^2\varphi}, \quad D_{0,\,n}^{s_1 s_1} = 0, \\ &A_{e,\,n}^{s_1 s_1} = \frac{n_e^4}{\varepsilon_e^2} - \left(\frac{n_e^4}{\varepsilon_e^2} - \frac{n_e^4}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi, \quad B_{e,\,n}^{s_1 s_1} = 2\frac{n_e^4}{\varepsilon_0^2}\frac{\sin^3\theta\cos\theta\sin^2\varphi\cos\varphi}{1-\sin^2\theta\sin^2\varphi}, \\ &C_{e,\,n}^{s_1 s_1} = -\frac{n_e^4}{\varepsilon_e^2} + \left(\frac{n_e^4}{\varepsilon_e^2} - \frac{n_e^4}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi + 2\frac{n_e^4}{\varepsilon_0^2}\frac{\sin^2\theta\cos^2\theta\sin^2\varphi}{1-\sin^2\theta\sin^2\varphi}, \\ &D_{e,\,n}^{s_1 s_1} = -2si\frac{n_e^4}{\varepsilon_0\varepsilon_e}\sin\theta\cos\theta\sin\varphi. \end{aligned}\right\} \tag{3.11}$$

相应的輻射強度为

$$W_{\perp,\,\lambda=0,\,e}^{s,\,s'} = W_{\perp,\,\lambda=0,\,e} + ss'W'_{\perp,\,\lambda=0,\,e}, \tag{3.12}$$

其中

$$\left.\begin{aligned} W_{\perp,\,0} = {} & \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\left[2\frac{\sin^2\theta\cos^2\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi} - 2\chi\frac{\omega\hbar}{cp\beta} + \right. \\ & \left. + (1-2\chi)\frac{\omega^2\hbar^2}{2c^2p^2}(\varepsilon_0 - 1)\right]\omega\,d\omega\,d\varphi, \\ W_{\perp,\,e} = {} & \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\left[2\frac{\varepsilon_e^2}{\varepsilon_0^2}\left\{\sin^2\theta\cos^2\theta\sin^2\varphi\Big/\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\times\right.\right. \\ & \left. \times\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right](\cos^2\varphi+\cos^2\theta\sin^2\varphi)\right\} + \\ & + 2\frac{\varepsilon_e^2}{\varepsilon_0^2}\frac{(\chi-1)\sin^2\theta\sin^2\varphi\,\dfrac{\omega\hbar}{cp\beta}}{\left[1-\left(1-\dfrac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\dfrac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]} + \\ & + \frac{2\dfrac{\varepsilon_e^2}{\varepsilon_0^2}\chi\sin^2\theta\sin^2\varphi + 1 - \left(1+\dfrac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi\,\omega^2\hbar^2}{\left[1-\left(1-\dfrac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\dfrac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]2c^2p^2}\times \\ & \left.\times\left(\frac{\varepsilon_e}{1-\left(1-\dfrac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi} - 1\right)\right]\omega\,d\omega\,d\varphi, \\ W'_{\perp,\,0} = {} & \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\frac{1}{\Gamma}\frac{\omega\hbar}{cp\beta}\left\{2\frac{\sin^2\theta\cos^2\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}\frac{cp\beta}{\omega\hbar} - 2\chi + \right. \\ & \left. + \left[(2\chi-1)\left(\frac{1}{\beta^2} - \frac{\varepsilon_0 - 1}{2}\right) + 1\right]\frac{\omega\hbar\beta}{cp} + (2\chi-1)\frac{\omega^2\hbar^2}{2c^2p^2}(\varepsilon_0 - 1)\right\}\omega\,d\omega\,d\varphi, \end{aligned}\right\} \tag{3.13}$$

$$
\left.\begin{aligned}
W'_{\perp,e} = {} & \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\frac{1}{\Gamma}\frac{\omega\hbar}{cp\beta}\times \\
& \times\left\{2\frac{\varepsilon_e^2}{\varepsilon_0^2}\frac{\sin^2\theta\sin^2\varphi(\chi-1-\cos^2\theta)+\dfrac{\sin^2\theta\cos^2\theta\sin^2\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}\dfrac{cp\beta}{\omega\hbar}}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}-\right. \\
& -\left\{\left[2\frac{\varepsilon_e^2}{\varepsilon_0^2}\chi\sin^2\theta\sin^2\varphi+1-\left(1+\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi\right]\left[\frac{1}{\beta^2}-\frac{1}{2}\times\right.\right. \\
& \times\left.\left.\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi}-1\right)\right]-1+\left(1+\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi\right\}\frac{\omega\hbar\beta}{cp}\Big/ \\
& \left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]\Bigg\}- \\
& -\frac{\left[2\frac{\varepsilon_e^2}{\varepsilon_0^2}\chi\sin^2\theta\sin^2\varphi+1-\left(1+\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi\right]\frac{\omega^2\hbar^2}{2c^2p^2}}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}\times \\
& \times\left.\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi}-1\right)\right\}\omega\,d\omega\,d\varphi,
\end{aligned}\right|
$$

式中 $\chi=\dfrac{\sin^2\theta\,(\cos^2\varphi+\cos\varphi)}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}$，频率积分的区域同样由 $\cos\theta_{0,e}=f(\omega)\leqslant 1$ 决定.

对自旋末态求和 $\sum\limits_{s'=+1}^{-1}W^{s,s'}_{\lambda=0,e}$，得到在垂直于光軸运动情况下的輻射量子表达式:

$$
\left.\begin{aligned}
W_{\perp,0} = {} & \frac{e^2}{2\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\left[\frac{\sin^2\theta\cos^2\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}-\chi\frac{\omega\hbar}{cp\beta}+\right. \\
& \left.+(1-2\chi)\frac{\omega^2\hbar^2}{4c^2p^2}(\varepsilon_0-1)\right]\omega\,d\omega\,d\varphi, \\
W_{\perp,e} = {} & \frac{e^2}{2\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\left\{\frac{\varepsilon_e^2}{\varepsilon_0^2}\times\right. \\
& \times\frac{\sin^2\theta\cos^2\theta\sin^2\varphi}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right](\cos^2\varphi+\cos^2\theta\sin^2\varphi)}+ \\
& +\frac{\varepsilon_e^2}{\varepsilon_0^2}\frac{(\chi-1)\sin^2\theta\sin^2\varphi\frac{\omega\hbar}{cp\beta}}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}+ \\
& +\frac{2\frac{\varepsilon_e^2}{\varepsilon_0^2}\chi\sin^2\theta\sin^2\varphi+1-\left(1+\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}\frac{\omega^2\hbar^2}{4c^2p^2}\times \\
& \times\left.\left(\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi}-1\right)\right\}\omega\,d\omega\,d\varphi.
\end{aligned}\right\} \quad (3.14)
$$

在此种情况下，閾能上的輻射将保持量子特点，即只有在自旋发生改变时（例如：反演 $S=1\to S'=-1$）輻射才是可能的，其相应的閾能值为

$$
\left.\begin{aligned}
W_{\perp,0} &= \frac{e^2}{c^2}\int_0^{\omega_{\max}} \frac{\omega^3\hbar^2}{4c^2p^2}(\varepsilon_0-1)d\omega,\\
W_{\perp,e} &= \frac{e^2}{2\pi c^2}\int_0^{\omega_{\max}} \frac{1}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi}\,\frac{\omega^3\hbar^2}{4c^2p^2}(\varepsilon_e-1)d\omega\, d\varphi.
\end{aligned}\right\}\tag{3.15}
$$

对于横向极化的粒子 $\left(C_1=\frac{1}{\sqrt{2}},\quad C_{-1}=\frac{1}{\sqrt{2}}e^{i\varphi_0}\right)$ 可得系数值

$$
\left.\begin{aligned}
&A_{0,n}^{s,s_1}=\frac{1}{2},\quad B_{0,n}^{s,s_1}=-\frac{\sin\theta\cos\theta\cos\varphi}{1-\sin^2\theta\sin^2\varphi},\quad C_{0,n}^{s,s_1}=\frac{1}{2}-\frac{\cos^2\theta}{1-\sin^2\theta\sin^2\varphi},\\
&D_{0,n}^{s,s_1}=0,\quad A_{0,n}^{-s,s_1}=\frac{1}{2}e^{is_1\varphi_0},\quad B_{0,n}^{-s,s_1}=-\frac{\sin\theta\cos\theta\cos\varphi}{1-\sin^2\theta\sin^2\varphi}e^{is_1\varphi_0},\\
&C_{0,n}^{-s,s_1}=\left(\frac{1}{2}-\frac{\cos^2\theta}{1-\sin^2\theta\sin^2\varphi}\right)e^{is_1\varphi_0},\quad D_{0,n}^{-s,s_1}=0;\\
&A_{e,n}^{s,s_1}=\frac{n_e^4}{\varepsilon_e^2}-\left(\frac{n_e^4}{\varepsilon_e^2}-\frac{n_e^4}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi,\quad B_{e,n}^{s,s_1}=\frac{n_e^4}{\varepsilon_0^2}\,\frac{\sin^3\theta\cos\theta\sin^2\varphi\cos\varphi}{1-\sin^2\theta\sin^2\varphi},\\
&C_{e,n}^{s,s_1}=\frac{1}{2}\left[-\frac{n_e^4}{\varepsilon_e^2}+\left(\frac{n_e^4}{\varepsilon_e^2}-\frac{n_e^4}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi+2\frac{n_e^4}{\varepsilon_0^2}\,\frac{\sin^2\theta\cos^2\theta\sin^2\varphi}{1-\sin^2\theta\sin^2\varphi}\right],\\
&D_{e,n}^{s,s_1}=-si\,\frac{n_e^4}{\varepsilon_0\varepsilon_e}\sin\theta\cos\theta\sin\varphi,\\
&A_{e,n}^{-s,s_1}=\frac{1}{2}\left[-\frac{n_e^4}{\varepsilon_e^2}+\left(\frac{n_e^4}{\varepsilon_0^2}+\frac{n_e^4}{\varepsilon_e^2}\right)\sin^2\theta\sin^2\varphi-2s_1i\,\frac{n_e^4}{\varepsilon_0\varepsilon_e}\sin^2\theta\sin\varphi\cos\varphi\right]e^{is_1\varphi_0},\\
&B_{e,n}^{-s,s_1}=\left[\frac{n_e^4}{\varepsilon_0^2}\,\frac{\sin^3\theta\cos\theta\sin^2\varphi\cos\varphi}{1-\sin^2\theta\sin^2\varphi}-s_1i\,\frac{n_e^4}{\varepsilon_0\varepsilon_e}\sin\theta\cos\theta\sin\varphi\right]e^{is_1\varphi_0},\\
&C_{e,n}^{-s,s_1}=\frac{1}{2}\left[\frac{n_e^4}{\varepsilon_e^2}-\left(\frac{n_e^4}{\varepsilon_0^2}+\frac{n_e^4}{\varepsilon_e^2}\right)\sin^2\theta\sin^2\varphi+2\frac{n_e^4}{\varepsilon_0^2}\,\frac{\sin^2\theta\cos^2\theta\sin^2\varphi}{1-\sin^2\theta\sin^2\varphi}+\right.\\
&\qquad\left.+2s_1i\frac{n_e^4}{\varepsilon_0\varepsilon_e}\sin^2\theta\sin\varphi\cos\varphi\right]e^{is_1\varphi_0},\\
&D_{e,n}^{-s,s_1}=0\qquad(S=S_1=\pm1).
\end{aligned}\right\}\tag{3.16}
$$

代入(2.13)得輻射強度为

$$
W_{\perp,\lambda=0,e}^{s'}=W_{\perp,\lambda=0,e}+s'W_{\perp,\lambda=0,e}'',\tag{3.17}
$$

和粒子的自旋状态改变相联系的量子补充項为

$$
\begin{aligned}
W_{\perp,0}'' &= \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\frac{1}{\Gamma}\,\frac{\omega^2\hbar}{cp\beta}\left\{2\,\frac{\cos\theta\cos\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}+\right.\\
&\quad\left.+\left[1-2\frac{\cos^2\theta(\cos\varphi+1)}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}\right]\frac{\omega\hbar}{cp}\sqrt{\varepsilon_0}\right\}\sqrt{1-\beta^2}\sin\theta\cos\varphi_0\,d\omega\,d\varphi,\\
W_{\perp,e}'' &= \frac{e^2}{8\pi c^2}\int_0^{\omega_{\max}}\int_0^{2\pi}\frac{1}{\Gamma}\,\frac{\omega^2\hbar}{cp\beta}\times
\end{aligned}
$$

$$\left.\begin{aligned}&\times\left\{\left[\frac{2\frac{\varepsilon_e^2}{\varepsilon_0^2}\left(\frac{\cos\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}-\chi\right)\sin^2\theta\sin^2\varphi+1-\left(1-\frac{\varepsilon_e^2}{\varepsilon_0^2}\right)\sin^2\theta\sin^2\varphi}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}\times\right.\right.\\&\times\frac{\omega\hbar}{cp}\sqrt{\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi}}-\\&-\left.\frac{2\frac{\varepsilon_e^2}{\varepsilon_0^2}\sin\theta\cos\theta\sin^2\varphi\cos\varphi}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right](\cos^2\varphi+\cos^2\theta\sin^2\varphi)}\right]\cos\varphi_0-\\&-2\frac{\varepsilon_e}{\varepsilon_0}\frac{\cos\theta\sin\varphi-(\cos^2\theta\sin\varphi-\sin^2\theta\sin\varphi\cos\varphi)}{\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi\right]\left[1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\varphi\right]}\times\\&\left.\times\frac{\omega\hbar}{cp}\sqrt{\frac{\varepsilon_e}{1-\left(1-\frac{\varepsilon_e}{\varepsilon_0}\right)\sin^2\theta\sin^2\varphi}}\sin\varphi_0\right\}\sqrt{1-\beta^2}\sin\theta\,d\omega\,d\varphi,\end{aligned}\right\}\quad(3.18)$$

在超相对論性近似 ($\beta\to 1$) 中，$W''_{\perp,\lambda=0,e}=0$. 在閾能 ($\cos\theta=1$) 上該項也将消失掉.

同样，对自旋末态求和 $\sum_{s'=1}^{-1}W^{s'}_{\perp,\lambda=0,e}$ 得垂直于光軸运动的輻射量子表达式(3.14).

在考虑到介质的各向异性时，輻射具有很复杂的形式，但有一点应指出，无論是經典的或量子的計算，寻常波輻射强度只决定于参量 ε_0, 在这个意义上講，常波輻射类似于各向同性介质中的輻射(如果将 $\varepsilon_0\to\varepsilon$)，复杂只是在非常波輻射.

若考虑到經典的輻射条件 $\cos\theta=\frac{1}{\beta n_{e,0}}$ 和等式 $\beta^2 n_e^2=\frac{\beta^2\varepsilon_0\varepsilon_e+(\varepsilon_e-\varepsilon_0)\sin^2\varphi}{\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\varphi}$，便得已知的各向异性介质中契連科夫輻射的經典表达式[1]($\hbar\to 0$)

$$\left.\begin{aligned}&W^{s,s'}_{\|,0}=W^{s'}_{\|,0}=0,\\&W^{s,s'}_{\|,e}=W^{s'}_{\|,e}=\frac{e^2}{c^2}\int_0^{\omega_{\max}}\left(1-\frac{1}{\beta^2\varepsilon_0}\right)\omega\,d\omega,\\&W^{s,s'}_{\perp,0}=W^{s'}_{\perp,0}=\frac{e^2}{2\pi c^2}\iint_0^{\omega_{\max}}\frac{(\beta^2\varepsilon_0-1)\cos^2\varphi}{\beta^2\varepsilon_0\cos^2\varphi+\sin^2\varphi}\omega\,d\omega\,d\varphi,\\&W^{s,s'}_{\perp,e}=W^{s'}_{\perp,e}=\frac{e^2}{2\pi c^2}\frac{1}{\beta^2}\iint_0^{\omega_{\max}}\frac{(\beta^2\varepsilon_e-1)\sin^2\varphi\omega\,d\omega\,d\varphi}{[\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\varphi][\beta^2\varepsilon_0\cos^2\varphi+\sin^2\varphi]},\end{aligned}\right\}\quad(3.19)$$

积分区域决定于 $v\geqslant\frac{c}{n_{0,e}(\omega)}$; φ 为 0—2π.

在这种情况下，(3.5)，(3.15)将等于零. 这說明在閾能上的輻射乃是純粹的量子效应(这仅仅对带电粒子而言，在磁矩的情况下，当运动垂直于光軸时，經典計算也得到閾能上的輻射值[12])，这是由于在量子理論中計入了粒子的自旋状态的結果[11].

四、无自旋粒子的輻射情况

为了进一步估計粒子的自旋对輻射的貢献，我們討論一下无自旋粒子的輻射情况．这时，粒子运动的波函数应取 Klein-Gordon 方程的解[11]：

$$\left.\begin{aligned} \psi &= L^{-3/2}\sum_{\mathbf{k}'} c b' e^{-icK't+i\mathbf{k}'\mathbf{r}}, \\ b_1' &= (2c\hbar K)^{-1/2};\quad b_2' = (c\hbar K/2)^{1/2}, \end{aligned}\right\} \tag{4.1}$$

而代替式(2.12)的矩陣元 $Q_\lambda^{+s,s'}\cdot Q_\lambda^{s,s'}$ 应写成

$$Q_\lambda^+ Q_\lambda = b^+(\boldsymbol{\alpha}^0\mathbf{a}_\lambda)b'\cdot b^{+\prime}(\boldsymbol{\alpha}^0\mathbf{a}_\lambda^+)b. \tag{4.2}$$

考虑到 $b\boldsymbol{\alpha}^0 = b\mathbf{k}/K$，从(2.12)可以得到以下的輻射强度表达式：

$$W_\lambda = \frac{e^2}{2\pi c^2}\int \frac{n_\lambda^4(\theta)}{k^2} f(\theta)(\mathbf{k}'\cdot\mathbf{a}_\lambda)(\mathbf{k}'\mathbf{a}_\lambda^+)\omega d\omega\, d\varphi. \tag{4.3}$$

这样，在粒子沿光軸运动的情况下，若选取运动方向与坐标系 z 軸重合，从 $\mathbf{k}\perp\mathbf{a}_0$ 便立刻得到 $W_{\|,0}=0$；从(2.6)算出相应的 $\boldsymbol{\beta}_e = \frac{\cos\theta}{\varepsilon_0}\mathbf{e}_x - \frac{\sin\theta}{\varepsilon_e}\mathbf{e}_z$ 代到(4.3)中，且考虑到 $\mathbf{a}_e\perp\mathbf{a}_0$ 便求出非常波的輻射强度表达式：

$$W_{\|,e} = \frac{e^2}{c^2}\int\left[\frac{\varepsilon_0 + \dfrac{\hbar\omega}{cp\beta}(\varepsilon_e-\varepsilon_0)}{\varepsilon_0-(\varepsilon_0-\varepsilon_e)\cos^2\theta} + \right.$$
$$\left. + \frac{\hbar^2\omega^2}{2c^2p^2}\frac{(\varepsilon_e-\varepsilon_0)}{\varepsilon_0-(\varepsilon_0-\varepsilon_e)\cos^2\theta}\left(\frac{\varepsilon_0\varepsilon_e}{\varepsilon_0-(\varepsilon_0-\varepsilon_e)\cos^2\theta}-1\right)\right]\sin^2\theta\,\omega\, d\omega. \tag{4.4}$$

这个結果正好說明，当粒子沿光軸运动时，上节中出現之常波輻射不为零的现象乃为一純粹的自旋效应．

当粒子垂直于光軸运动时，式(2.6)在 $\mathbf{j}^0$ 平行于 y 軸的情况下变成

$$\left.\begin{aligned} \boldsymbol{\beta}_0 &= \frac{1}{\varepsilon_0}\frac{-\cos\theta\,\mathbf{e}_x + \sin\theta\cos\varphi\,\mathbf{e}_z}{\sqrt{1-\sin^2\theta\sin^2\varphi}}, \\ \boldsymbol{\beta}_e &= \frac{1}{\sqrt{1-\sin^2\theta\sin^2\varphi}}\left[\frac{1}{\varepsilon_0}\sin^2\theta\sin\varphi\cos\varphi\mathbf{e}_x + \right. \\ &\quad \left. + \frac{1}{\varepsilon_e}(\sin^2\theta\sin^2\varphi-1)\mathbf{e}_y + \frac{1}{\varepsilon_0}\sin\theta\cos\theta\sin\varphi\,\mathbf{e}_z\right]. \end{aligned}\right\} \tag{4.5}$$

相应的寻常波和非常波輻射强度将为

$$\left.\begin{aligned} W_{\perp,0} &= \frac{e^2}{2\pi c^2}\iint\frac{\sin^2\theta\cos^2\varphi}{\cos^2\varphi+\cos^2\theta\sin^2\varphi}\,\omega\, d\omega\, d\varphi, \\ W_{\perp,e} &= \frac{e^2}{2\pi c^2}\iint\Big\{[\varepsilon_e^2\sin^2\theta\cos^2\theta\sin^2\varphi/[\cos^2\varphi+\cos^2\theta\sin^2\varphi][\varepsilon_0 - \\ &\quad -(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi][\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\varphi]] + \\ &\quad + \frac{\hbar\omega}{cp\beta}\frac{\varepsilon_e(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi}{[\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi][\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\varphi]} + \\ &\quad + \frac{\hbar^2\omega^2}{2c^2p^2}\frac{\varepsilon_e(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi}{[\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi][\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\varphi]}\times \\ &\quad \times\left(\frac{\varepsilon_0\varepsilon_e}{\varepsilon_0-(\varepsilon_0-\varepsilon_e)\sin^2\theta\sin^2\varphi}-1\right)\Big\}\,\omega\, d\omega\, d\varphi. \end{aligned}\right\} \tag{4.6}$$

从輻射能的表达式(4.4),(4.6)可以看出,在各向异性介质中,輻射的量子理論有别于各向同性介質中的情况[12],对于无自旋粒子的輻射已出現量子效应(获得量子补充項),而在閾能上($\cos\theta = 1$),該部与經典項一起为零.

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КВАНТОВАЯ ТЕОРИЯ ЧЕРЕНКОВСКОГО ИЗЛУЧЕНИЯ В АНИЗОТРОПНЫХ СРЕДАХ

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Резюме

На основе квантования электромагнитного поля рассмотрено черенковское излучение в анизотропных средах. Получены интенсивности излучения, соответствующие как обыкновенной волне, так и необыкновенной волне. Выяснено влияние спина частиц на интенсивности излучения. Квантовые расчёты отличаются от классических при учёте спина частиц. В случае движения по оптической оси имеется излучение как обыкновеной волны, так и необыкновенной волны.

Даны результаты анализа в двух случаях: движение по оптической оси и по направлению, перпендикулярному оптической оси.