Sup-norm convergence rate and sign concentration property of Lasso and Dantzig estimators

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Abstract

We derive the l_∞ convergence rate simultaneously for Lasso and Dantzig estimators in a high-dimensional linear regression model under a mutual coherence assumption on the Gram matrix of the design and two different assumptions on the noise: Gaussian noise and general noise with finite variance. Then we prove that simultaneously the thresholded Lasso and Dantzig estimators with a proper choice of the threshold enjoy a sign concentration property provided that the non-zero components of the target vector are not too small.

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