Weighted least squares methods for prediction in the functional data linear model

Aurore Delaigle, *University of Bristol and University of Melbourne*
Peter Hall, *University of Melbourne and University of California at Davis*
Tatiyana V. Apanasovich, *Thomas Jefferson University*

Abstract

The problem of prediction in functional linear regression is conventionally addressed by reducing dimension via the standard principal component basis. In this paper we show that weighted least-squares methods can be more effective when the experimental errors are heteroscedastic. We give a concise theoretical result which demonstrates the effectiveness of this approach, even when the model for the variance is inaccurate, and we explore the numerical properties of the method. We show too that the advantages of the suggested adaptive techniques are not found only in low-dimensional aspects of the problem; rather, they accrue almost equally among all dimensions.

Full Text: PDF

References

Besse, P. and Ramsay J.O. (1986). Principal components-analysis of sampled functions. Psychometrika 51, 285–311. MR0848110

Boente, G. and Fraiman, R. (2000). Kernel-based functional principal components. Statist. Probab. Lett. 48, 335–345. MR1771495

Brumback, B.A. and Rice, J.A. (1998). Smoothing spline models for the analysis of nested and crossed samples of curves. J. Amer. Statist. Assoc. 93, 961–976. MR1649194

Cai, T.T. and Hall, P. (2006). Prediction in functional linear regression. Ann. Statist. 34, 2159–2179. MR2291496

Cardot, H. (2000). Nonparametric estimation of smoothed principal components analysis of sampled noisy functions. J. Nonparam. Statist. 12, 503–538. MR1785396

Cardot, H., Crambes, C. Kneip, A. and Sarda, P. (2006). Smoothing splines estimators in functional linear regression with errors-in-variables. Comput. Statist. Data Anal. 51, 4832–4848. MR2364543

Cardot, H., Ferraty, F. and Sarda, P. (1999). Functional linear model. Statist. Probab. Lett. 45, 11–22. MR1718346

Cardot, H., Ferraty, F. and Sarda, P. (2000). Étude asymptotique d'un estimateur spline hybride pour le modèle linéaire fonctionnel. C.R. Acad. Sci. Paris Sér. I 330, 501–504. MR1756966

Cardot, H., Ferraty, F. and Sarda, P. (2003). Spline estimators for the functional linear model. Statist. Sinica 13, 571–591. MR1997162

Cardot, H. and Sarda, P. (2006). Linear regression models for functional data. In: The Art of Semiparametrics Eds. S. Sperlich, W. Härdle and G. Aydinli, pp. 49–66. Springer, Berlin. MR2234875

Carroll, R. and Ruppert, D. (1988). Transformation and weighting in regression. Chapman and Hall, New York. MR1014890

Cuevas, A., Febrero, M. and Fraiman, R. (2002). Linear functional regression: the case of fixed design and functional response. Canad. J. Statist. 30, 285–300. MR1926066

Faraway, J.J. (1997). Regression analysis for a functional response. Technometrics 39, 254–261. MR1462586

Ferraty, F. and Vieu, P. (2000). Fractal dimensionality and regression estimation in semi-normed vectorial spaces. C.R. Acad. Sci. Paris Sér. I 330, 139–142. MR1745172

Ferraty, F. and Vieu, P. (2006). Nonparametric Functional Data Analysis. Theory and Practice. Springer, New York. MR2229687

Girard, S. (2000). A nonlinear PCA based on manifold approximation. Comput. Statist. 15, 145–167. MR1794107

Hall, P. and Hosseini-Nasab, M. (2006). On properties of functional principal components analysis. J. R. Stat. Soc. Ser. B 68, 109–126. MR2212577

Hall, P. and Hosseini-Nasab, M. (2009). Theory for high-order bounds in functional principal components analysis. Math. Proc. Camb. Phil. Soc. 146, 225–256. MR2461880

He, G.Z., Müller, H.-G. and Wang, J.-L. (2003). Functional canonical analysis for square integrable stochastic processes. J. Multivar. Anal. 85, 54–77. MR1978177

Huang, J.Z., Shen, H. and Buja, A. (2008). Functional principal components analysis via penalized rank one approximation. Electron. J. Stat. 2, 678–695. MR2426107

James, G.M., Hastie, T.J. and Sugar, C.A. (2000). Principal component models for sparse functional data. Biometrika 87, 587–602. MR1789811

Jank, W. and Shmueli, G. (2006). Functional data analysis in electronic commerce research. Statist. Sci. 21, 155–166. MR2324075

Lavery, B., Kariko, A. and Nicholls, N. (1992). A historical rainfall data set for Australia. Aust. Met. Mag., 40, 33–39.

Mas, A. (2008). Local functional principal component analysis. Complex Anal. Oper. Theory 2, 135–167. MR2390677

Ocaña, F.A., Aguilera, A.M. and Escabias, M. (2007). Computational considerations in functional principal component analysis. Comput. Statist. 22, 449–465. MR2336347

Ramsay, J.O. and Dalzell, C.J. (1991). Some tools for functional data analysis. (With discussion.) J. Roy. Statist. Soc. Ser. B 53, 539–572. MR1125714

Ramsay, J.O. and Silverman, B.W. (2005). Functional Data Analysis., 2nd Edn. Springer, New York. MR2168993

Reiss, P.T. and Ogden, T. (2007). Functional principal component regression and functional partial least squares. J. Amer. Statist. Assoc. 102, 984–996. MR2411660

Rice, J.A. and Silverman, B.W. (1991). Estimating the mean and covariance structure nonparametrically when the data are curves. J. Roy. Statist. Soc. Ser. B 53, 233–243. MR1094283

Silverman, B.W. (1995). Incorporating parametric effects into functional principal components analysis. J. Roy. Statist. Soc. Ser. B 57, 673–689. MR1354074

Silverman, B.W. (1996). Smoothed functional principal components analysis by choice of norm. Ann. Statist. 24, 1–24. MR1389877

Yao, F., Müller, H.-G. and Wang, J.-L. (2005). Functional linear regression analysis for longitudinal data. Ann. Statist. 33, 2873–2903. MR2253106