Almost sure convergence of extreme order statistics

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Abstract

Let $M_{n}^{(k)}$ denote the kth largest maximum of a sample $(X_{1}, X_{2}, \ldots, X_{n})$ from parent X with continuous distribution. Assume there exist normalizing constants $a_{n} > 0$, $b_{n} \in \mathbb{R}$ and a nondegenerate distribution G such that $a_{n}^{-1}(M_{n}^{(1)} - b_{n}) \stackrel{w}{\to} G$. Then for fixed $k \in \mathbb{N}$, the almost sure convergence of

$$\frac{1}{D_{N}}\sum_{n=k}^{N}d_{n} \mathbb{I}\{M_{n}^{(1)}\le a_{n}x_{1}+b_{n}, M_{n}^{(2)} \le a_{n}x_{2}+ b_{n}, \ldots, M_{n}^{(k)}\le a_{n}x_{k} + b_{n}\}$$

is derived if the positive weight sequence (d_{n}) with $D_{N} = \sum_{n=1}^{N}d_{n}$ satisfies conditions provided by H\"{o}rmann. Some practical issues of this result are also discussed.

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