Functional asymptotic confidence intervals for a common mean of independent random variables

Yuliya V. Martsynyuk, *Carleton University and University of Ottawa*

Abstract

We consider independent random variables (r.v.'s) with a common mean μ that either satisfy Lindeberg's condition, or are symmetric around μ. Present forms of existing functional central limit theorems (FCLT's) for Studentized partial sums of such r.v.'s on $D[0,1]$ are seen to be of some use for constructing asymptotic confidence intervals, or what we call functional asymptotic confidence intervals (FACI's), for μ. In this paper we establish completely data-based versions of these FCLT's and thus extend their applicability in this regard. Two special examples of new FACI's for μ are presented.

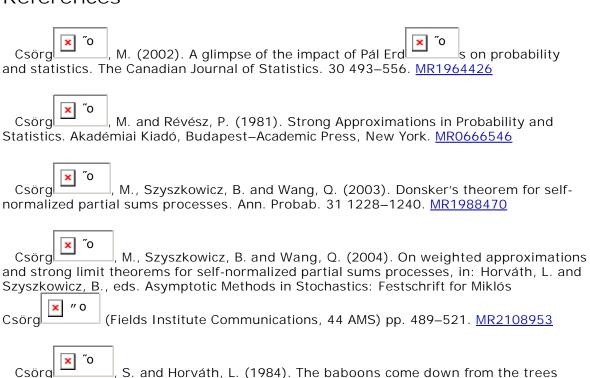AMS 2000 subject classifications: Primary 60F17, 60G50, 62G15.

Keywords: Lindeberg's condition, symmetric random variable, Student statistic, Student process, Wiener process, functional central limit theorem, sup-norm approximation in probability, functional asymptotic confidence interval.

Full Text: PDF

References

Csörgő, M. (2002). A glimpse of the impact of Pál Erdős on probability and statistics. The Canadian Journal of Statistics. 30 493–556. MR1964426

Csörgő, M. and Révész, P. (1981). Strong Approximations in Probability and Statistics. Akadémiai Kiadó, Budapest–Academic Press, New York. MR0666546

Csörgő, M., Szyszkowicz, B. and Wang, Q. (2003). Donsker's theorem for self-normalized partial sums processes. Ann. Probab. 31 1228–1240. MR1988470

Csörgő, M., Szyszkowicz, B. and Wang, Q. (2004). On weighted approximations and strong limit theorems for self-normalized partial sums processes, in: Horváth, L. and Szyszkowicz, B., eds. Asymptotic Methods in Stochastics: Festschrift for Miklós Csörgő (Fields Institute Communications, 44 AMS) pp. 489–521. MR2108953

Csörgő, S. and Horváth, L. (1984). The baboons come down from the trees quite normally, in: Proceedings of the 4th Pannonian Symposium on Mathematical Statistics (Akadémiai Kiadó, Budapest) pp. 95–106. MR0851047

Egorov, V.A. (1996). On the asymptotic behavior of self-normalized sums of random variables. Teor. Veroyatnost. i Primenen. 41 643–650. MR1450081

Gnedenko, B.V. and Kolmogorov, A.N. (1954). Limit Distributions for Sums of Independent Random Variables. Addison-Wesley, Reading, MA. MR0062975

Graybill, F.A. and Deal, R.B. (1959). Combining unbiased estimators. Biometrics. 15 543–550. MR0107925

Martsynyuk, Yu.V. (2008). Functional asymptotic confidence intervals for the slope in linear error-in-variables models. Acta Math. Hungar. To appear.

Norwood, T.E.Jr. and Hinkelmann, K. (1977). Estimating the common mean of several normal populations. Ann. Statist. 5 1047–1050. MR0448679

Pal, N. and Lim, W.K. (1997). A note on second-order admissibility of the Graybill-Deal estimator of a common mean of several normal populations. J. Statist. Plann. Inference. 63 71–78. MR1474186

Prohorov, Yu. V. (1956). Convergence of random processes and limit theorems in probability theory. Teor. Veroyatnost. i Primenen. 1 157–214. MR0084896

Račkauskas, A. and Suquet, Ch. (2001). Invariance principles for adaptive self-normalized partial sums processes. Stochastic Process. Appl. 95 63–81. MR1847092

Shorack, G.R. (2000). Probability for Statisticians. Springer-Verlag, New York. MR1762415

"Student" (1908). The probable error of a mean. Biometrika. 6 1–25.