Functional limit laws for the increments of the quantile process; with applications

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Abstract

We establish a functional limit law of the logarithm for the increments of the normed sample quantile process based upon random sample of size $n->infty$. We extend a limit law obtained by Deheuvels and Mason (1992), showing that their results hold uniformly over the bandwidth h, restricted to vary in $[h'_n,h''_n]$, where ${h'_n}_{ngeq1}$ and ${h''_n}_{ngeq1}$ are appropriate non-random sequences. We treat the case where the sample observations follow possibly non-uniform distributions. As a consequence of our theorems, we provide uniform limit laws for nearest-neighbor density estimators, in the same spirit as those given by Deheuvels and Mason (2004) for kernel-type estimators.

AMS 2000 subject classifications: Primary 60F15, 60F17; secondary 62G07.

Keywords: functional limit laws, laws of the iterated logarithm, empirical process, quantile process, order statistics, density estimation, nonparametric estimation, nearest-neighbor estimates.

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