

## Controlling for individual heterogeneity in longitudinal models, with applications to student achievement

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### Abstract

Longitudinal data tracking repeated measurements on individuals are highly valued for research because they offer controls for unmeasured individual heterogeneity that might otherwise bias results. Random effects or mixed models approaches, which treat individual heterogeneity as part of the model error term and use generalized least squares to estimate model parameters, are often criticized because correlation between unobserved individual effects and other model variables can lead to biased and inconsistent parameter estimates. Starting with an examination of the relationship between random effects and fixed effects estimators in the standard unobserved effects model, this article demonstrates through analysis and simulation that the mixed model approach has a "bias compression" property under a general model for individual heterogeneity that can mitigate bias due to uncontrolled differences among individuals. The general model is motivated by the complexities of longitudinal student achievement measures, but the results have broad applicability to longitudinal modeling.

Keywords: longitudinal data analysis, random effects models, fixed effects models, omitted variables.



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