Parameter estimation of ODE's via nonparametric estimators

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Abstract
Ordinary differential equations (ODE's) are widespread models in physics, chemistry and biology. In particular, this mathematical formalism is used for describing the evolution of complex systems and it might consist of high-dimensional sets of coupled nonlinear differential equations. In this setting, we propose a general method for estimating the parameters indexing ODE's from times series. Our method is able to alleviate the computational difficulties encountered by the classical parametric methods. These difficulties are due to the implicit definition of the model. We propose the use of a nonparametric estimator of regression functions as a first-step in the construction of an M-estimator, and we show the consistency of the derived estimator under general conditions. In the case of spline estimators, we prove asymptotic normality, and that the rate of convergence is the usual $\sqrt{n}$-rate for parametric estimators. Some perspectives of refinements of this new family of parametric estimators are given.

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