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Asymptotic properties of false discovery rate controlling procedures under independence

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Abstract

We investigate the performance of a family of multiple comparison procedures for strong control of the False Discovery Rate (FDR). The FDR is the expected False Discovery Proportion (FDP), that is, the expected fraction of false rejections among all rejected hypotheses. A number of refinements to the original Benjamini-Hochberg procedure [1] have been proposed, to increase power by estimating the proportion of true null hypotheses, either implicitly, leading to one-stage adaptive procedures [4, 7] or explicitly, leading to two-stage adaptive (or plug-in) procedures [2, 21].

We use a variant of the stochastic process approach proposed by Genovese and Wasserman [11] to study the fluctuations of the FDP achieved with each of these procedures around its expectation, for independent tested hypotheses.

We introduce a framework for the derivation of generic Central Limit Theorems for the FDP of these procedures, characterizing the associated regularity conditions, and comparing the asymptotic power of the various procedures. We interpret recently proposed one-stage adaptive procedures [4, 7] as fixed points in the iteration of well known two-stage adaptive procedures [2, 21].

AMS 2000 subject classifications: 62G10, 62H15, 60F05.

Keywords: Multiple hypothesis testing, Benjamini-Hochberg procedure, FDP, FDR.

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