Nonparametric deconvolution problem for dependent sequences

Rafał Kulik, *University of Ottawa*

Abstract

We consider the nonparametric estimation of the density function of weakly and strongly dependent processes with noisy observations. We show that in the ordinary smooth case the optimal bandwidth choice can be influenced by long range dependence, as opposite to the standard case, when no noise is present. In particular, if the dependence is moderate the bandwidth, the rates of mean-square convergence and, additionally, central limit theorem are the same as in the i.i.d. case. If the dependence is strong enough, then the bandwidth choice is influenced by the strength of dependence, which is different when compared to the non-noisy case. Also, central limit theorem are influenced by the strength of dependence. On the other hand, if the density is supersmooth, then long range dependence has no effect at all on the optimal bandwidth choice.

AMS 2000 subject classifications: Primary 62G05; secondary 62G07, 60F05.

Keywords: long range dependence, linear processes, error-in-variables models, deconvolution.

Full Text: PDF

Kulik, Rafał, Nonparametric deconvolution problem for dependent sequences, Electronic Journal of Statistics, 2, (2008), 722-740 (electronic). DOI: 10.1214/07-EJS154.

References

[1] Beran, J. and Feng, Y. (2001). Local polynomial estimation with a FARIMA-GARCH error process. Bernoulli, 7, 733–750. MR1867080

[2] Beran, J. and Feng, Y. (2001). Local polynomial fitting with long-memory, short-memory and antipersistent errors. Ann. Inst. Statist. Math., 54, 291–311. MR1910174

[3] Bosq, D. (1996). Nonparametric Statistics for Stochastic Processes. Lecture Notes in Statistics 110. Springer, New York. MR1441072

[4] Carroll, R.J. and Hall, P. (1988). Optimal Rates of Convergence for Deconvoling a Density. J. Amer. Statist. Assoc. 83, 1184–1186. MR0997599

[5] Claeskens, G. and Hall, P. (2002). Effect of dependence on stochastic measures of accuracy of density estimators. Ann. Statist. 30, 431–454. MR1902894

[6] Cs˝orgö, S. and Mielniczuk, J. (1995). Density estimation under long-range dependence. Ann. Statist. 23, 990–999. MR1345210

[7] Doukhan, P. (1984). Mixing: Properties and Examples. Lecture Notes in Statisitcs. Springer. MR1312160

[8] Estevez, G. and Vieu, P. (2003). Nonparametric estimation under long memory

dependence. Nonparametirc Statistics 15, 535–551. MR2017486

[9] Fan, J. (1991). On the optimal rates of convergence for nonparametric deconvolution problems. Ann. Statist. 19, 1257–1272. MR1126324

[10] Fan, J. (1991). Asymptotic normality for deconvolving kernel density estimators. Sankya Ser. A 53, 97–110. MR1177770

[11] Fan, J. and Masry, E. (1992). Multivariate Regression Estimation with Errors-in-Variables: Asymptotic Normality for Mixing Processes. J. Mult. Anal. 43, 237–271. MR1193614

[12] Giraitis, L. and Surgailis, D. (1999). Central limit theorem for the empirical process of a linear sequence with long memory. J. Statist. Plann. Inference 80, 81–93. MR1713796

[13] Hall, P. and Hart, J.D. (1990). Convergence rates in density estimation for data from infinite-order moving average processes. Probab. Th. Rel. Fields 87, 253–274. MR1080492

[14] Hall, P., Lahiri, S. N. and Truong, Y. K. (1995). On bandwidth choice for density estimation with dependent data. Ann. Statist. 23, 2241–2263. MR1389873

[15] Ho, H.-C. and Hsing, T. (1996). On the asymptotic expansion of the empirical process of long-memory moving averages. Ann. Statist. 24, 992–1024. MR1401834

[16] Ho, H.-C. and Hsing, T. (1997). Limit theorems for functionals of moving averages. Ann. Probab. 25, 1636–1669. MR1487431

[17] Ioannides, D.A. and Papanastassiou, D.P. (2001). Estimating the Distribution Function of a Stationary Process Involving Measurement Errors. Statist. Inf. for Stoch. Proc. 4, 181–198. MR1856173

[18] Masry, E. (1991). Multivariate Probability Density Deconvolution for Stationary Random Processes. IEEE Trans. Inf. Th. 37, 1105–1115. MR1111811

[19] Masry, E. (1993). Strong consistency and rates for deconvolution of multivariate densities of stationary processes. Stochastic Process. Appl. 47, 53–74. MR1232852

[20] Masry, E. (1993). Asymptotic normality for deconvolving estimators of multivariate densities for stationary processes. J. Mult. Anal. 44, 47–68. MR1208469

[21] Masry, E. (2003). Deconvolving Multivariate Kernel Density Estimates From Contaminated Associated Observations. IEEE Trans. Inf. Th. 49, 2941–2952. MR2027570

[22] Masry, E. and Mielniczuk, J. (1999). Local linear regression estimation for time series with long-range dependence. Stoch. Proc. Appl. 82, 173–193. MR1700004

[23] Mielniczuk, J. (1997). On the asymptotic mean integrated squared error of a kernel density estimator for dependent data. Statist. Probab. Letters 34, 53–58. MR1457496

[24] Mielniczuk, J. and Wu, W. B. (2004). On random-design model with dependent errors. Statistica Sinica, 1105–1126. MR2126343

[25] Stefanski, L.A. and Carroll, R.J. (1990). Deconvoltuing kernel density estimators. Statistics 21, 169–184. MR1054861

[26] Wu, W.B. (2003). Empirical processes of long-memory sequences. Bernoulli 9, 809–831. MR2047687

[27] Wu, W.B. (2005). On the Bahadur representation of sample quantiles for dependent sequences. Ann. Statist. 33, 1934–1963. MR2166566

[28] Wu, W. B. and Mielniczuk, J. (2002) Kernel density estimation for linear processes. Ann. Statist. 30, 1441–1459. MR1936325

Electronic Journal of Statistics. ISSN: 1935-7524