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Structural shrinkage of nonparametric spectral estimators for multivariate time series

Hilmar Böhm, *Institut de statistique, UC Louvain*
Rainer von Sachs, *Institut de statistique, UC Louvain*

Abstract

In this paper we investigate the performance of periodogram based estimators of the spectral density matrix of possibly high-dimensional time series. We suggest and study shrinkage as a remedy against nu- merical instabilities due to deteriorating condition numbers of (kernel) smoothed periodogram matrices. Moreover, shrinking the empirical eigenvalues in the frequency domain towards one another also improves at the same time the Mean Squared Error (MSE) of these widely used nonparametric spectral estimators. Compared to some existing time domain approaches, restricted to i.i.d. data, in the frequency domain it is necessary to take the size of the smoothing span as "effective or local sample size" into account. While Böhm & von Sachs (2007) pro- poses a multiple of the identity matrix as optimal shrinkage target in the absence of knowledge about the multidimensional structure of the data, here we consider "structural" shrinkage. We assume that the spectral structure of the data is induced by underlying factors. How- ever, in contrast to actual factor modelling suffering from the need to choose the number of factors, we suggest a model-free approach. Our final estimator is the asymptotically MSE-optimal linear combination of the smoothed periodogram and the parametric estimator based on an undefitting (and hence deliberately misspecified) factor model. We complete our theoretical considerations by some extensive simulation studies. In the situation of data generated from a higher-order factor model, we compare all four types of involved estimators (including the one of Böhm & von Sachs (2007)).

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