

## Adaptive complexity regularization for linear inverse problems

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### Abstract

We tackle the problem of building adaptive estimation procedures for ill-posed inverse problems. For general regularization methods depending on tuning parameters, we construct a penalized method that selects the optimal smoothing sequence without prior knowledge of the regularity of the function to be estimated. We provide for such estimators oracle inequalities and optimal rates of convergence. This penalized approach is applied to Tikhonov regularization and to regularization by projection.

AMS 2000 subject classifications: Primary 62G05, 34K29.

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