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On the asymptotic properties of the group lasso estimator for linear models

Yuval Nardi, *Department of Statistics, Carnegie Mellon University*
Alessandro Rinaldo, *Department of Statistics, Carnegie Mellon University*

Abstract

We establish estimation and model selection consistency, prediction and estimation bounds and persistence for the group-lasso estimator and model selector proposed by Yuan and Lin (2006) for least squares problems when the covariates have a natural grouping structure. We consider the case of a fixed-dimensional parameter space with increasing sample size and the double asymptotic scenario where the model complexity changes with the sample size.

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