

**CORRECTION ON
 MOMENTS OF MINORS OF WISHART MATRICES**

Ann. Statist. **36** (2008) 2261–2283

BY MATHIAS DRTON AND ALDO GOIA

University of Chicago and University of East Piedmont

Theorem 5.7 in [1] gives a formula for the variance of a minor (i.e., a sub-determinant) of a Wishart random matrix. The formula has to be corrected as follows.

THEOREM 5.7. *Let $I, J \in \binom{[r]}{m}$ have intersection $C := I \cap J$ of cardinality $c = |C| = |I \cap J|$. Define $\bar{I} = I \setminus (I \cap J)$, $\bar{J} = J \setminus (I \cap J)$ and $\bar{I}\bar{J} = \bar{I} \cup \bar{J}$. Then the minor $\det(S_{I \times J}) = \det(S_{IJ})$ of the Wishart matrix $S \sim \mathcal{W}_r(n, \Sigma)$ has variance*

$$\begin{aligned} \text{Var}[\det(S_{IJ})] &= \det(\Sigma_{IJ})^2 \frac{n!}{(n-m)!} \left[\frac{(n+2)!}{(n+2-m)!} - \frac{n!}{(n-m)!} \right] \\ &\quad + \det(\Sigma_{C \times C})^2 \det(\bar{\Sigma}_{\bar{I}\bar{J} \times \bar{I}\bar{J}}) \frac{(n+2)!}{(n+2-c)!} \cdot \frac{n!}{(n-m)!} \\ &\quad \times \left[\sum_{k=0}^{m-c-1} (m-c-k)! \frac{(n+2-c)!}{(n+2-c-k)!} (-1)^k \text{tr}\{(\bar{\Sigma}_{\bar{I}\bar{J}} \bar{\Sigma}_{\bar{I}\bar{J}})^{(k)}\} \right], \end{aligned}$$

where $\bar{\Sigma} = \Sigma_{([r] \setminus C) \times ([r] \setminus C)} - \Sigma_{([r] \setminus C) \times C} \Sigma_{C \times C}^{-1} \Sigma_{C \times ([r] \setminus C)}$.

PROOF. Define \bar{S} in analogy to $\bar{\Sigma}$. Since $\det(S_{IJ}) = \det(S_{C \times C}) \det(\bar{S}_{\bar{I} \times \bar{J}})$, and $S_{C \times C}$ and $\bar{S}_{\bar{I} \times \bar{J}}$ are independent (Lemma 5.2),

$$\begin{aligned} \text{Var}[\det(S_{IJ})] &= (\text{Var}[\det(S_{CC})] + \text{E}[\det(S_{CC})]^2) \text{Var}[\det(\bar{S}_{\bar{I}\bar{J}})] \\ &\quad + \text{Var}[\det(S_{CC})] \text{E}[\det(\bar{S}_{\bar{I}\bar{J}})]^2. \end{aligned}$$

Received March 2012.

AMS 2000 subject classifications. 60E05, 62H10.

Key words and phrases. Compound matrix, graphical models, multivariate analysis, random determinant, random matrix, tetrad.

This is an electronic reprint of the original article published by the [Institute of Mathematical Statistics](#) in *The Annals of Statistics*, 2012, Vol. 40, No. 2, 1283–1284. This reprint differs from the original in pagination and typographic detail.

The claim now follows from Corollary 4.2 and Propositions 5.1 and 5.5. \square

REFERENCE

- [1] DRTON, M., MASSAM, H. and OLKIN, I. (2008). Moments of minors of Wishart matrices. *Ann. Statist.* **36** 2261–2283. [MR2458187](#)

DEPARTMENT OF STATISTICS
UNIVERSITY OF CHICAGO
CHICAGO, ILLINOIS 60637
USA
E-MAIL: drton@galton.uchicago.edu

DIPARTIMENTO DI STUDI PER L'ECONOMIA
E L'IMPRESA
UNIVERSITY OF EAST PIEDMONT
NOVARA
ITALY
E-MAIL: aldo.goia@eco.unipmn.it