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The Law of the Hitting Times to Points by a Stable Lévy Process with No Negative Jumps

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Abstract

Let $X=(X_t)_{t ge 0}$ be a stable L'evy process of index $alpha in (1,2)$ with the L'evy measure $nu(dx) = (c/x^{1+alpha}):! I_{(0,infty)}(x), dx$ for $c>0$, let $x>0$ be given and fixed, and let $tau_x = inf, {, t>0 : X_t=x, }$ denote the first hitting time of X to x. Then the density function f_{tau_x} of tau_x admits the following series representation: begin{align} notag f_{tau_x}(t) = frac{x^{alpha-1}}{pi:! (c;! Gamma(-alpha) ;!t)^{2-1/alpha}} sum_{n=1}^infty bigg[:! & (-1)^{n-1} sin(pi/alpha), frac{Gamma(n!-!1/alpha)}{Gamma(alpha n!-!1)}, Big(frac{x^alpha}{c;! Gamma(-alpha);!t} Big)^{!n-1} notag &- sinBig(frac{n pi}{alpha} Big), frac{Gamma(1!+!n/alpha)}{n!}, Big(frac{x^alpha}{c;! Gamma(-alpha);!t}Big)^{!(n+1)/alpha-1}, bigg] end{align} for $t>0$. In particular, this yields $f_{tau_x}(0+)=0$ and begin{equation} notag f_{tau_x}(t) sim frac{x^{alpha-1}}{Gamma(alpha!-!1), Gamma(1/alpha)}, (c;! Gamma(-alpha);!t)^{-2+1/alpha} end{equation} as $t rightarrow infty$. The method of proof exploits a simple identity linking the law of tau_x to the laws of X_t and $sup_{,0 le s le t} X_s$ that makes a Laplace inversion amenable. A simpler series representation for f_{tau_x} is also known to be valid when $x<0$.

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