

## Large and Moderate Deviations for Hotelling's $T^2$ -Statistics

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### Abstract

Let  $X, X_1, X_2, \dots$  be i.i.d.  $\mathbb{R}^d$ -valued random variables. We prove large and moderate deviations for Hotelling's  $T^2$ -statistic when  $X$  is in the generalized domain of attraction of the normal law.

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