



# Simple, asymptotically distribution-free, optimal tests for circular reflective symmetry about a known median direction

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In this paper, we propose optimal tests for circular reflective symmetry about a fixed median direction. The distributions against which optimality is achieved are the so-called k-sine-skewed distributions of Umbach and Jammalamadaka (2009). We first show that sequences of k-sine-skewed models are locally and asymptotically normal in the vicinity of reflective symmetry. Following the Le Cam methodology, we then construct optimal (in the maximin sense) parametric tests for reflective symmetry, which we render semi-parametric by a studentization argument. These asymptotically distribution-free tests happen to be uniformly optimal (under any reference density) and are moreover of a very simple and intuitive form. They furthermore exhibit nice small sample properties, as we show through a Monte Carlo simulation study. Our new tests also allow us to re-visit the famous red wood ants data set of Jander (1957). We further show that one of the proposed parametric tests can as well serve as a test for uniformity against cardioid alternatives; this test coincides with the famous circular Rayleigh (1919) test for uniformity which is thus proved to be (also) optimal against cardioid alternatives. Moreover, our choice of k-sine-skewed alternatives, which are the circular analogues of the classical linear skew-symmetric distributions, permits us a Fisher singularity analysis à la Hallin and Ley (2012) with the result that only the prominent sine-skewed von Mises distribution suffers from these inferential drawbacks. Finally, we conclude the paper by discussing the unspecified location case.

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