

## Fixed Points of the Smoothing Transform: the Boundary Case

John D Biggins, *University of Sheffield*  
Andreas E Kyprianou, *Heriot-Watt University*

### Abstract

Let  $A=(A_1, A_2, A_3, \dots)$  be a random sequence of non-negative numbers that are ultimately zero with  $E[\sum A_i]=1$  and  $E\left[\sum A_{\{i\}} \log A_i\right] \leq 0$ . The uniqueness of the non-negative fixed points of the associated smoothing transform is considered. These fixed points are solutions to the functional equation  $\Phi(\psi) = E\left[\prod_{\{i\}} \Phi(\psi A_i)\right]$ , where  $\Phi$  is the Laplace transform of a non-negative random variable. The study complements, and extends, existing results on the case when  $E\left[\sum A_{\{i\}} \log A_i\right] < 0$ . New results on the asymptotic behaviour of the solutions near zero in the boundary case, where  $E\left[\sum A_{\{i\}} \log A_i\right] = 0$ , are obtained.

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