

Some remarks on tangent martingale difference sequences in L^1 -spaces

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Abstract

Let X be a Banach space. Suppose that for all p in $(1, \infty)$ a constant $C_{p,X}$ depending only on X and p exists such that for any two X -valued martingales f and g with tangent martingale difference sequences one has

$$E\|f\|^p \leq C_{p,X} E\|g\|^p \quad (*).$$

This property is equivalent to the UMD condition. In fact, it is still equivalent to the UMD condition if in addition one demands that either f or g satisfy the so-called (CI) condition. However, for some applications it suffices to assume that (*) holds whenever g satisfies the (CI) condition. We show that the class of Banach spaces for which (*) holds whenever only g satisfies the (CI) condition is more general than the class of UMD spaces, in particular it includes the space L^1 . We state several problems related to (*) and other decoupling inequalities.

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