On the lower bound of the spectral norm of symmetric random matrices with independent entries

Sandrine Peche, *Institut Fourier, Grenoble, France*
Alexander Soshnikov, *University of California at Davis, USA*

Abstract

We show that the spectral radius of an N-dimensional random symmetric matrix with i.i.d. bounded centered but non-symmetrically distributed entries is bounded from below by $2\sigma - o(N^{-6/11+\varepsilon})$, where σ^2 is the the variance of the matrix entries and $\varepsilon > 0$ is an arbitrary small positive number. Combining with our previous result from [7], this proves that for any $\varepsilon > 0$ one has $||A_N|| = 2\sigma + o(N^{-6/11+\varepsilon})$ with probability going to 1 as N goes to infinity.

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