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Explicit Bounds for the Approximation Error in Benford's Law

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Abstract

Benford's law states that for many random variables $X > 0$ its leading digit $D = D(X)$ satisfies approximately the equation $\mathbb{P}(D = d) = \log_{10}(1 + 1/d)$ for $d = 1,2,\ldots,9$. This phenomenon follows from another, maybe more intuitive fact, applied to $Y := \log_{10} X$: For many real random variables Y, the remainder $U := Y - \lfloor Y \rfloor$ is approximately uniformly distributed on $[0,1)$. The present paper provides new explicit bounds for the latter approximation in terms of the total variation of the density of Y or some derivative of it. These bounds are an interesting and powerful alternative to Fourier methods. As a by-product we obtain explicit bounds for the approximation error in Benford's law.

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