

## Large and Moderate Deviations for Hotelling's $T^2$ -Statistics

Amir Dembo, *Stanford University*

Qi-Man Shao, *Hong Kong University of Science and Technology*

### Abstract

Let  $X, X_1, X_2, \dots$  be i.i.d.  $\mathbb{R}^d$ -valued random variables. We prove large and moderate deviations for Hotelling's  $T^2$ -statistic when  $X$  is in the generalized domain of attraction of the normal law.

Full text: [PDF](#) | [PostScript](#)

Pages: 149-159

Published on: August 7, 2006

### Bibliography

1. Anderson, T.W. (1984). *An introduction to Multivariate Analysis* (2nd ed.). Wiley, New York. [MR0771294 \(86b:62079\)](#)
2. Bercu, B., Gassiat, E. and Rio, E. (2002). Concentration inequalities, large and moderate deviations for self-normalized empirical processes. *Ann. Probab.* 30, 1576--1604. [MR1944001 \(2004a:60060\)](#)
3. Bingham, N. H., Goldie, C. M. and Teugels, J. L. (1987). *Regular variation*. Cambridge University Press, Cambridge. [MR0898871 \(88i:26004\)](#)
4. Chernoff, H. (1952). A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations. *Ann. Math. Stat.* 23, 493-507. [MR0057518 \(15,241c\)](#)
5. Chistyakov, G.P. and Gotze, F. (2004a). On bounds for moderate deviations for Student's statistic. *Theory Probab. Appl.* 48, 528-535. [MR2141355 \(2005m:60092\)](#)
6. Chistyakov, G.P. and Gotze, F. (2004b). Limit distributions of Studentized means. *Ann. Probab.* 32 (2004), 28-77. [MR2040775 \(2005f:60055\)](#)
7. Dembo, A. and Shao, Q.M. (1998a). Self-normalized moderate deviations and lils. *Stoch. Proc. and Appl.* 75, 51-65. [MR1629018 \(99i:60053\)](#)
8. Dembo, A. and Shao, Q.M. (1998b). Self-normalized large deviations in vector spaces. In: *Progress in Probability* (Eberlein, Hahn, Talagrand, eds) Vol. 43, 27-32. [MR1652318 \(99j:60036\)](#)
9. Faure, M. (2002). Self-normalized large deviations for Markov chains. *Electronic J. Probab.* 7, 1-31. [MR1943896 \(2003j:60034\)](#)
10. Fujikoshi, Y. (1997). An asymptotic expansion for the distribution of Hotelling's  $T^2$ -statistic under nonnormality. *J. Multivariate Anal.* 61, 187-193. [MR1452272 \(98h:62091\)](#)
11. Griffin, P. and Kuelbs, J. (1989). Self-normalized laws of the iterated logarithm. *Ann. Probab.* 17, 1571--1601. [MR1048947 \(91k:60036\)](#)
12. Hahn, M.G. and Klass, M.J. (1980). Matrix normalization of sums of random vectors in the domain of attraction of the multivariate normal. *Ann. Probab.* 8, 262-280. [MR0566593 \(81d:60030\)](#)
13. He, X. and Shao, Q. M. (1996). Bahadur efficiency and robustness of studentized score tests. *Ann. Inst. Statist. Math.* 48, 295-314. [MR1405934 \(97m:62016\)](#)
14. Jing, B.Y., Shao, Q.M. and Wang, Q.Y. (2003). Self-normalized Cramer type large deviations for independent random variables. *Ann. Probab.* 31, 2167-2215. [MR1999791 \(2004f:60046\)](#)
15. Jing, B.Y., Shao, Q.M. and Zhou, W. (2004). Saddlepoint approximation for Student's t-statistic with no moment conditions. *Ann. Statist.* 32, 2679-2711. [MR2153999 \(2006a:62030\)](#)
16. Kano, Y. (1995). An asymptotic expansion of the distribution of Hotelling's  $T^2$ -statistic under general condition. *Amer. J. Math. Manage. Sci.* 15, 317-341. [MR1397511 \(97h:62044\)](#)

### Research Support Tool

[Capture Cite](#)  
[View Metadata](#)  
[Supplementary Files and / or CORRECTIONS](#)  
[Printer Friendly](#)

▼ [Context](#)

[Author Address](#)

▼ [Action](#)

[Email Others](#)

17. Kariya, T. (1981). A robustness property of Hotelling's  $T^2$ -test. *Ann. Statist.* 9, 210-213. [MR0600550 \(82c:62052\)](#)
18. Kiefer, J. and Schwartz, R. (1965). Admissible Bayes character of  $T^2$ - and  $R^2$ - and other fully invariant tests for classical normal problems. *Ann. Math. Statist.* 36, 747-760. [MR0175245 \(30 #5430\)](#)
19. Muirhead, R.J. (1982). *Aspects of Multivariate Statistical Theory*. John Wiley, New York. [MR0652932 \(84c:62073\)](#)
20. Robinson, J. and Wang, Q.Y. (2005). On the self-normalized Cramer-type large deviation. *J. Theoretic Probab.* 18, 891-909.
21. Sepanski, S. (1994). Asymptotics for Multivariate t-statistic and Hotelling's  $T^2$ -statistic under infinite second moments via bootstrapping. *J. Multivariate Anal.* 49, 41-54. [MR1275042 \(96d:62092\)](#)
22. Shao, Q.M. (1997). Self-normalized large deviations. *Ann. Probab.* 25, 285--328. [MR1428510 \(98b:60056\)](#)
23. Shao, Q.M. (1998). Recent developments in self-normalized limit theorems. In *Asymptotic Methods in Probability and Statistics* (editor B. Szyszkowicz), pp. 467 - 480. Elsevier Science. [MR1661499 \(99j:60063\)](#)
24. Shao, Q.M. (2004). Recent progress on self-normalized limit theorems. In *Probability, finance and insurance* (editors Tze Leung Lai, Hailiang Yang and Siu Pang Yung), pp. 50--68, World Sci. Publ., River Edge, NJ, 2004. [MR2189198 \(2006i:60056\)](#)
25. Simaika, J.B. (1941). On an optimal property of two important statistical tests. *Biometrika* 32, 70-80. [MR0003547 \(2,236b\)](#)
26. Stein, C. (1956). The admissibility of Hotelling's T-test. *Ann. Math. Statist.* 27, 616-623. [MR0080413 \(18,243c\)](#)
27. Wang, Q.Y. (2005). Limit theorems for self-normalized large deviation. *Electronic J. Probab.* 10, 1260-1285. [MR2176384](#)

