Spherical and Hyperbolic Fractional Brownian Motion

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Abstract
We define a Fractional Brownian Motion indexed by a sphere, or more generally by a compact rank one symmetric space, and prove that it exists if, and only if, $0 < H \leq 1/2$. We then prove that Fractional Brownian Motion indexed by an hyperbolic space exists if, and only if, $0 < H \leq 1/2$. At last, we prove that Fractional Brownian Motion indexed by a real tree exists when $0 < H \leq 1/2$.

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