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An elementary proof of Hawkes's conjecture on Galton-Watson trees.

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Abstract

In 1981, J. Hawkes conjectured the exact form of the Hausdorff gauge function for the boundary of supercritical Galton-Watson trees under a certain assumption on the tail at infinity of the total mass of the branching measure. Hawkes's conjecture has been proved by T. Watanabe in 2007 as well as other precise results on fractal properties of the boundary of Galton-Watson trees. The goal of this paper is to provide an elementary proof of Hawkes's conjecture under a less restrictive assumption than in T. Watanabe's paper, by use of size-biased Galton-Watson trees introduced by Lyons, Pemantle and Peres in 1995.

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