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Covariance Estimation for Distributions with $2+\epsilon$ Moments

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We study the minimal sample size $N=N(n)$ that suffices to estimate the covariance matrix of an n -dimensional distribution by the sample covariance matrix in the operator norm, and with an arbitrary fixed accuracy. We establish the optimal bound $N = O(n)$ for every distribution whose k -dimensional marginals have uniformly bounded $2+\epsilon$ moments outside the sphere of radius $O(\sqrt{k})$. In the specific case of log-concave distributions, this result provides an alternative approach to the Kannan-Lovasz-Simonovits problem, which was recently solved by Adamczak, Litvak, Pajor and Tomczak-Jaegermann. Moreover, a lower estimate on the covariance matrix holds under a weaker assumption -- uniformly bounded $2+\epsilon$ moments of one-dimensional marginals. Our argument proceeds by randomizing the spectral sparsification technique of Batson, Spielman and Srivastava. The spectral edges of the sample covariance matrix are controlled via the Stieltjes transform evaluated at carefully chosen random points.

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