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Sharp maximal inequality for martingales and stochastic integrals

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Abstract

Let $X=(X_t)_{tgeq 0}$ be a martingale and $H=(H_t)_{tgeq 0}$ be a predictable process taking values in $[-1,1]$. Let Y denote the stochastic integral of H with respect to X. We show that $$ ||sup_{tgeq 0}Y_t||_1 leq beta_0 ||sup_{tgeq 0}|X_t|||_1,$$ where $beta_0=2,0856ldots$ is the best possible. Furthermore, if, in addition, X is nonnegative, then $$ ||sup_{tgeq 0}Y_t||_1 leq beta_0^+ ||sup_{tgeq 0}X_t||_1,$$ where $beta_0^+=frac{14}{9}$ is the best possible.

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