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When uniform weak convergence fails: empirical processes for dependence functions via epi- and hypographs

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(Submitted on 28 May 2013)

For copulas whose partial derivatives are not continuous everywhere on the interior of the unit cube, the empirical copula process does not converge weakly with respect to the supremum distance. This makes it hard to verify asymptotic properties of inference procedures for such copulas. To resolve the issue, a new metric for locally bounded functions is introduced and the corresponding weak convergence theory is developed. Convergence with respect to the new metric is related to epi- and hypoconvergence and is weaker than uniform convergence. Still, for continuous limits, it is equivalent to locally uniform convergence, whereas under mild side conditions, it implies L^p convergence. Even in cases where uniform convergence fails, weak convergence with respect to the new metric is established for empirical copula and tail dependence processes. No additional assumptions are needed for tail dependence functions, and for copulas, the assumptions reduce to existence and continuity of the partial derivatives almost everywhere on the unit cube. The results are applied to obtain asymptotic properties of minimum distance estimators, goodness-of-fit tests and resampling procedures.

Comments:	44 pages
Subjects:	**Statistics Theory (math.ST)**
MSC classes:	Primary 60F05, 62G30, secondary 62G32, 62M09
Cite as:	**arXiv:1305.6408 [math.ST]**

(or **arXiv:1305.6408v1 [math.ST]** for this