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Calculation of Exact Estimators by Integration Over the Surface of an n-Dimensional Sphere

Anthony J Webster

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This paper reconsiders the problem of calculating the expected set of probabilities <p_i>, given the observed set of items {m_i}, that are distributed among n bins with an (unknown) set of probabilities {p_i} for being placed in the ith bin. The problem is often formulated using Bayes theorem and the multinomial distribution, along with a constant prior for the values of the p_i, leading to a Dirichlet distribution for the {p_i}. The moments of the p_i can then be calculated exactly. Here a new approach is suggested for the calculation of the moments, that uses a change of variables that reduces the problem to an integration over a portion of the surface of an n-dimensional sphere. This greatly simplifies the calculation by allowing a straightforward integration over (n-1) independent variables, with the constraints on the set of p_i being automatically satisfied. For the Dirichlet and similar distributions the problem simplifies even further, with the resulting integrals subsequently factorising, allowing their easy evaluation in terms of Beta functions. A proof by induction confirms existing calculations for the moments. The advantage of the approach presented here is that the methods and results apply with minimum or no modifications to numerical calculations that involve more complicated distributions or non-constant prior distributions, for which cases the numerical calculations will be greatly simplified.

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