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A framework to characterize performance of LASSO algorithms

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(Submitted on 29 Mar 2013)

In this paper we consider solving \emph{noisy} under-determined systems of linear equations with sparse solutions. A noiseless equivalent attracted enormous attention in recent years, above all, due to work of \cite{CRT,CanRomTao06,DonohoPol} where it was shown in a statistical and large dimensional context that a sparse unknown vector (of sparsity proportional to the length of the vector) can be recovered from an under-determined system via a simple polynomial ℓ_1-optimization algorithm. \cite{CanRomTao06} further established that even when the equations are \emph{noisy}, one can, through an SOCP noisy equivalent of ℓ_1, obtain an approximate solution that is (in an ℓ_2-norm sense) no further than a constant times the noise from the sparse unknown vector. In our recent works \cite{StojnicCSetam09,StojnicUpper10}, we created a powerful mechanism that helped us characterize exactly the performance of ℓ_1 optimization in the noiseless case (as shown in \cite{StojnicEquiv10} and as it must be if the axioms of mathematics are well set, the results of \cite{StojnicCSetam09,StojnicUpper10} are in an absolute agreement with the corresponding exact ones from \cite{DonohoPol}). In this paper we design a mechanism, as powerful as those from \cite{StojnicCSetam09,StojnicUpper10}, that can handle the analysis of a LASSO type of algorithm (and many others) that can be (or typically are) used for "solving" noisy under-determined systems. Using the mechanism we then, in a statistical context, compute the exact worst-case ℓ_2 norm distance between the unknown sparse vector and the approximate one obtained through such a LASSO. The obtained results match the corresponding exact ones obtained in \cite{BayMon10,DonMalMon10}. Moreover, as a by-product of our analysis framework we recognize existence of an SOCP type of algorithm that achieves the same performance.

Subjects: **Information Theory (cs.IT)**; Optimization and Control (math.OC); Probability (math.PR); Statistics Theory (math.ST)

Cite as: **arXiv:1303.7291 [cs.IT]**
(or **arXiv:1303.7291v1 [cs.IT]** for this version)

Submission history

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