

A Characterization of the Set of Fixed Points of the Quicksort Transformation

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Abstract

The limiting distribution m of the normalized number of key comparisons required by the Quicksort sorting algorithm is known to be the unique fixed point of a certain distributional transformation T -- unique, that is, subject to the constraints of zero mean and finite variance. We show that a distribution is a fixed point of T if and only if it is the convolution of m with a Cauchy distribution of arbitrary center and scale. In particular, therefore, m is the unique fixed point of T having zero mean.

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Published on: May 26, 2000

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