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A Representation for Non-Colliding Random Walks

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Abstract

We define a sequence of mappings $Gamma_k:D_0(R_+)^kto D_0(R_+)^k$ and prove the following result: Let $N_1,ldots,N_n$ be the counting functions of independent Poisson processes on R_+ with respective intensities $mu_1<mu_2<cdots <mu_n$. The conditional law of $N_1,ldots,N_n$, given that $$N_1(t)lecdotsle N_n(t), mbox{ for all }tge 0,$$ is the same as the unconditional law of $Gamma_n(N)$. From this, we deduce the corresponding results for independent Poisson processes of equal rates and for independent Brownian motions (in both of these cases the conditioning is in the sense of Doob). This extends a recent observation, independently due to Baryshnikov (2001) and Gravner, Tracy and Widom (2001), which relates the law of a certain functional of Brownian motion to that of the largest eigenvalue of a GUE random matrix. Our main result can also be regarded as a generalisation of Pitman's representation for the 3-dimensional Bessel process.

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