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Ergodicity of PCA: Equivalence between Spatial and Temporal Mixing Conditions

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Abstract

For a general attractive Probabilistic Cellular Automata on S^{Z^d}, we prove that the (time-) convergence towards equilibrium of this Markovian parallel dynamics, exponentially fast in the uniform norm, is equivalent to a condition A. This condition means the exponential decay of the influence from the boundary for the invariant measures of the system restricted to finite boxes.

For a class of reversible PCA dynamics on $\{-1;+1\}^{Z^d}$ with a naturally associated Gibbsian potential φ, we prove that a (spatial-) weak mixing condition WM for φ implies the validity of the assumption A ; thus *exponential (time-) ergodicity* of these dynamics towards the unique Gibbs measure associated to φ holds. On some particular examples we state that exponential ergodicity holds as soon as there is no phase transition.

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