

## An Extreme-Value Analysis of the LIL for Brownian Motion

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### Abstract

We use excursion theory and the ergodic theorem to present an extreme-value analysis of the classical law of the iterated logarithm (LIL) for Brownian motion. A simplified version of our method also proves, in a paragraph, the classical theorem of Darling and Erdős (1956).

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