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On the spectral norm of a random Toeplitz matrix

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Abstract

Suppose that T_n is a Toeplitz matrix whose entries come from a sequence of independent but not necessarily identically distributed random variables with mean zero. Under some additional tail conditions, we show that the spectral norm of T_n is of the order $\sqrt{(n \log n)}$. The same result holds for random Hankel matrices as well as other variants of random Toeplitz matrices which have been studied in the literature.

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