

Recurrence for branching Markov chains

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Abstract

The question of recurrence and transience of branching Markov chains is more subtle than for ordinary Markov chains; they can be classified in transience, weak recurrence, and strong recurrence. We review criteria for transience and weak recurrence and give several new conditions for weak recurrence and strong recurrence. These conditions make a unified treatment of known and new examples possible and provide enough information to distinguish between weak and strong recurrence. This represents a step towards a general classification of branching Markov chains. In particular, we show that in homogeneous cases weak recurrence and strong recurrence coincide. Furthermore, we discuss the generalization of positive and null recurrence to branching Markov chains and show that branching random walks on \mathbb{Z}^d are either transient or positive recurrent.

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Pages: 576-605

Published on: November 24, 2008

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