Wigner theorems for random matrices with dependent entries: Ensembles associated to symmetric spaces and sample covariance matrices

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Abstract
It is a classical result of Wigner that for an hermitian matrix with independent entries on and above the diagonal, the mean empirical eigenvalue distribution converges weakly to the semicircle law as matrix size tends to infinity. In this paper, we prove analogs of Wigner's theorem for random matrices taken from all infinitesimal versions of classical symmetric spaces. This is a class of models which contains those studied by Wigner and Dyson, along with seven others arising in condensed matter physics. Like Wigner's, our results are universal in that they only depend on certain assumptions about the moments of the matrix entries, but not on the specifics of their distributions. What is more, we allow for a certain amount of dependence among the matrix entries, in the spirit of a recent generalization of Wigner's theorem, due to Schenker and Schulz-Baldes. As a byproduct, we obtain a universality result for sample covariance matrices with dependent entries.

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