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Spectral gap for the interchange process in a box

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Abstract

We show that the spectral gap for the interchange process (and the symmetric exclusion process) in a d-dimensional box of side length L is asymptotic to π^2/L^2. This gives more evidence in favor of Aldous's conjecture that in any graph the spectral gap for the interchange process is the same as the spectral gap for a corresponding continuous-time random walk. Our proof uses a technique that is similar to that used by Handjani and Jungreis, who proved that Aldous's conjecture holds when the graph is a tree.

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