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A Regeneration Proof of the Central Limit Theorem for Uniformly Ergodic Markov Chains

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Abstract

Central limit theorems for functionals of general state space Markov chains are of crucial importance in sensible implementation of Markov chain Monte Carlo algorithms as well as of vital theoretical interest. Different approaches to proving this type of results under diverse assumptions led to a large variety of CLT versions. However due to the recent development of the regeneration theory of Markov chains, many classical CLTs can be reproved using this intuitive probabilistic approach, avoiding technicalities of original proofs. In this paper we provide a characterization of CLTs for ergodic Markov chains via regeneration and then use the result to solve the open problem posed in [Roberts & Rosenthal 2005]. We then discuss the difference between one-step and multiple-step small set condition.

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