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PAC learnability under non-atomic measures: a problem by Vidyasagar

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In response to a 1997 problem of M. Vidyasagar, we state a criterion for PAC learnability of a concept class $\mathscr C$ under the family of all non-atomic (diffuse) measures on the domain Ω. The uniform Glivenko--Cantelli property with respect to non-atomic measures is no longer a necessary condition, and consistent learnability cannot in general be expected. Our criterion is stated in terms of a combinatorial parameter $\VC({\mathscr C}\,{\mathrm{mod}}\,\omega_1)$ which we call the VC dimension of $\mathscr C$ modulo countable sets. The new parameter is obtained by "thickening up" single points in the definition of VC dimension to uncountable "clusters". Equivalently, $\VC(\mathscr C\modd\omega_1)\leq d$ if and only if every countable subclass of $\mathscr C$ has VC dimension $\leq d$ outside a countable subset of Ω. The new parameter can be also expressed as the classical VC dimension of $\mathscr C$ calculated on a suitable subset of a compactification of Ω. We do not make any measurability assumptions on $\mathscr C$, assuming instead the validity of Martin's Axiom (MA). Similar results are obtained for function learning in terms of fat-shattering dimension modulo countable sets, but, just like in the classical distribution-free case, the finiteness of this parameter is sufficient but not necessary for PAC learnability under non-atomic measures.

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