Wavelet methods in statistics: Some recent developments and their applications

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Abstract

The development of wavelet theory has in recent years spawned applications in signal processing, in fast algorithms for integral transforms, and in image and function representation methods. This last application has stimulated interest in wavelet applications to statistics and to the analysis of experimental data, with many successes in the efficient analysis, processing, and compression of noisy signals and images. This is a selective review article that attempts to synthesize some recent work on ``nonlinear'' wavelet methods in nonparametric curve estimation and their role on a variety of applications. After a short introduction to wavelet theory, we discuss in detail several wavelet shrinkage and wavelet thresholding estimators, scattered in the literature and developed, under more or less standard settings, to denoise data modeled as observations of a signal with additive noise. Most of these methods are fitted into the general concept of regularization with appropriately chosen penalty functions. A narrow range of applications in major areas of statistics is also discussed such as partial linear regression models and functional index models. The usefulness of all these methods are illustrated by means of simulations and practical examples.

Full Text: PDF

Antoniadis, Anestis, Wavelet methods in statistics: Some recent developments and their applications, Statistics Surveys, , (), 16-55 (electronic). DOI: 10.1214/07-SS014.

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