Fast Computation of Optimal Contact Forces

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IEEE Transactions on Robotics, 23(6):1117-1132, December 2007.

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We consider the problem of computing the smallest contact forces, with point-contact friction model, that can hold an object in equilibrium against a known external applied force and torque. It is known that the force optimization problem (FOP) can be formulated as a semidefinite programming problem (SDP), or a second-order cone problem (SOCP), and so can be solved using several standard algorithms for these problem classes. In this paper we describe a custom interior-point algorithm for solving the FOP that exploits the specific structure of the problem, and is much faster than these standard methods. Our method has a complexity that is linear in the number of contact forces, whereas methods based on generic SDP or SOCP algorithms have complexity that is cubic in the number of forces. Our method is also much faster for smaller problems. We derive a compact dual problem for the FOP, which allows us to rapidly compute lower bounds on the minimum contact force, and to certify infeasibility of a FOP. We use this dual problem to terminate our optimization method with a guaranteed accuracy. Finally, we consider the problem of solving a family of FOPs that are related. This occurs, for example, in determining whether force closure occurs, in analyzing the worst-case contact force required over a set of external forces and torques, and in the problem of choosing contact points on an object so as to minimize the required contact force. Using dual bounds, and a warm-start version of our FOP method, we show how such families of FOPs can be solved very efficiently.