

Shannon Meets Lyapunov: Connections between Information Theory and Dynamical Systems

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This paper explores connections between Information Theory, Lyapunov exponents for products of random matrices, and hidden Markov models. Specifically, we will show that entropies associated with finite-state channels are equivalent to Lyapunov exponents. We use this result to show that the traditional prediction filter for hidden Markov models is not an irreducible Markov chain in our problem framework. Hence, we do not have access to many well-known properties of irreducible continuous state space Markov chains (e.g. a unique and continuous stationary distribution). However, by exploiting the connection between entropy and Lyapunov exponents and applying proof techniques from the theory of random matrix products we can solve a broad class of problems related to capacity and hidden Markov models. Our results provide strong regularity results for the non-irreducible prediction filter as well as some novel theoretical tools to address problems in these areas.