On Lyapunov Inequalities and Subsolutions for Efficient Importance Sampling

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In this article we explain some connections between Lyapunov methods and subsolutions of an associated Isaacs equation for the design of efficient importance sampling schemes. As we shall see, subsolutions can be derived by taking an appropriate limit of an associated Lyapunov inequality. They have been recently proposed in several works of Dupuis, Wang, and others and applied to address several important problems in rare-event simulation. Lyapunov inequalities have been used for testing the efficiency of state-dependent importance sampling schemes in heavy-tailed or discrete settings in a variety of works by Blanchet, Glynn, and others. While subsolutions provide an analytic criterion for the construction of efficient samplers, Lyapunov inequalities are useful for finding more precise information, in the form of bounds, for the behavior of the coefficient of variation of the associated importance sampling estimator in the prelimit. In addition, Lyapunov inequalities provide insight into the various mollification procedures that are often required in constructing associated subsolutions. Our aim is to demonstrate that applying Lyapunov inequalities for verification of efficiency can help both guide the selection of various mollification parameters and sharpen the information on the efficiency gain induced by the sampler.