On Convergence to Stationarity of Fractional Brownian Storage

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With $M(t):= \sup_{0\leq s\leq t} A(s)-s$ denoting the running maximum of a fractional Brownian motion $A(\cdot)$ with negative drift, this paper studies the rate of convergence of $P(M(t)>x)$ to $P(M>x)$. We define two metrics that measure the distance between the (complementary) distribution functions $P(M(t)>\cdot)$ and $P(M>\cdot)$. Our main result states that both metrics roughly decay as $\exp(-\&\#x03B8t^{2-2H})$, where &#x03B8 is the decay rate corresponding to the tail distribution of the busy period in an fBm-driven queue, which was computed recently [Stochastic Process. Appl. (2006) 116 1269–1293]. The proofs extensively rely on application of the well-known large deviations theorem for Gaussian processes. We also show that the identified relation between the decay of the convergence metrics and busy-period asymptotics holds in other settings as well, most notably when Gärtner–Ellis-type conditions are fulfilled.