Computing the Distribution Function of a Conditional Expectation via Monte Carlo: Discrete Conditioning Spaces

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We examine different ways of numerically computing the distribution function of conditional expectations where the conditioning element takes values in a finite or countably infinite outcome space. Both the conditional expectation and the distribution function itself are computed via Monte Carlo simulation. Given a limited (and fixed) computer budget, the quality of the estimator is gauged by the inverse of its mean square error. It is a function of the fraction of the budget allocated to estimating the conditional expectation versus the amount of sampling done relative to the "conditioning variable." We will present the asymptotically optimal rates of convergence for different estimators and resolve the trade-off between the bias and variance of the estimators. Moreover, central limit theorems are established for some of the estimators proposed. We will also provide algorithms for the practical implementation of two of the estimators and illustrate how confidence intervals can be formed in each case. Major potential application areas include calculation of Value at Risk (VaR) in the field of mathematical finance and Bayesian performance analysis.