Understanding PHMI for Safety of life applications in GNSS

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ABSTRACT

Future satellite navigation systems (like Galileo) intend to provide safety-of-life services, or at least a form of integrity information. Integrity is characterized by the Probability of Hazardously Misleading Information, which is the probability that a user might exceed a certain error level (either fixed are variable). This definition, however, is incomplete, as it does not specify what is assumed to be known when computing this probability. The goal of this work is to offer an interpretation of the PHMI, point out some of the difficulties encountered in the proof of safety, and finally suggest possible future changes in the system design to mitigate these difficulties.

INTRODUCTION

As opposed to GPS, the next generation of satellite navigation systems (Galileo[1], GPS III) intend to provide integrity information to users, so that the signals can be used for safety-of life applications. Integrity is usually measured by the Probability of Hazardously Misleading Information (PHMI) [2], which is defined as the probability that the information contained in the navigation message leads to a position error larger than a certain error bound (known by the user). Unfortunately, this definition is open to several interpretations. For example, this definition does not specify what is observable when computing the PHMI, nor does it say whether it applies to a worst case user or an average user. These different interpretations can have a very large impact on the actual level of performance.

To date, the only certified system providing integrity information for vertical guidance is the Wide Area Augmentation System (WAAS), which has been in operation since 2003. In this work we would like to give a more precise interpretation of the Probability of Hazardously Misleading Information that has been used for WAAS. One of the consequences of this approach is that, because we are dealing with such small probabilities, the PHMI can only be computed using a model for the failure modes and their probabilities (threat models).

We begin by offering an interpretation of the Probability of Hazardously Misleading Information in the context of GNSS and Satellite Based Augmentation Systems in particular. Next, we will highlight the difficulties in the proof of safety and examine their causes. In particular, we would like to identify whether the causes are unavoidable (lack of stationary data, insufficient data points) or are due to constraints arising from the system design (often dictated by the available bandwidth). Finally, we present ideas that might be useful for future GNSS standards for integrity (be it SBAS, GBAS, Galileo or GPS III).

INTERPRETATION OF THE PROBABILITY OF HAZARDOUSLY MISLEADING INFORMATION

As with any probability, it is extremely important to have a clear idea of what is supposed to be known when computing it, that is, on what the PHMI is conditioned on. Often, when discussing error characterization for the PHMI, possible disagreement stems from the definition that one has assumed. In particular one needs to define:

- criteria under which an error can be treated as random so that it can be convolved with other random events
- which classes of errors can be aggregated in order to model the data

It should be noted that the previous remarks imply that PHMI is *subjective* to a certain degree. In the case of WAAS, it was decided that the PHMI should not be averaged over conditions that are predictable or detectable in a practical way. Also, the PHMI could not be averaged over conditions that are unknown but constant or repeatable over time. As an example, here are some conditions over which it was decided the probability could not be averaged:

– receiver type (even though the specific behavior of each receiver is unknown)
– ionospheric conditions (disturbed vs nominal)
– location
– geometry

On the other hand, there is information that is not used to condition the PHMI. For example, the PHMI is not conditioned on the particular ranging measurements of a given user. Also, for some conditions, the answer is more complex. The idea here is that if there is a good reason that errors might depend on a condition, then the samples cannot be pooled together. If the condition is continuous, then the data might need to be pooled in bins where one can assume the condition uniform over it.

DIFFICULTIES IN THE PROOF OF SAFETY

Overbounding range errors

In the current SBAS scheme, the principal difficulty in the proof of safety was overbounding the range error distributions [3], both in the range domain and the position domain. It is important to point out from the start that the overbounding operation is actually composed of two distinct operations. The first operation is to obtain a representation of the error distribution that is conservative. This representation can be a histogram, a maximum error corresponding to a worst case for a given probability, or a Gaussian distribution with an undetermined (but bounded) mean. At this point, it does not need to be a practical representation. The second operation is to take that representation and replace it (conservatively) by a representation that is practical and such that it behaves adequately through convolution. In SBAS, a zero mean Gaussian representation is used. This last operation is a mathematical operation that has mostly been solved through different approaches [3], [4], [5]. It is essential not to confuse these two operations conceptually, even though they are often treated simultaneously.

The first operation is the more challenging one. It is possible to divide it arbitrarily into two main issues: overbounding the tails (extreme errors) and overbounding the core (possible biases in nominal situations). For the tails (guaranteed up to very small probabilities, on the order of 10^{-7}, in the case of WAAS range errors), the fundamental problem is the lack of data under all conditions. There is no way one can, for each error source, gather enough stationary or nearly stationary data to attain the required levels of confidence. Also, all errors are usually mixed together and it is not always possible to isolate one from the other. In overbounding the core, the difficulty lies in the fact that users might experience error biases in a repeatable pattern. It is not acceptable to assume a zero mean error distribution for this kind of error. Instead, the service provider needs to protect the worst case user. Unfortunately, these biases can be very difficult to observe [6] as they may be common mode to all WAAS measurements, but affect users differently.

The chosen method in the WAAS for both the tails and the mean of the distribution is to characterize the tails of the distribution through analysis [2]. However, to do that, one must hypothesize the mechanism of the threat and constitute a threat model. This is a critical step, and is, to a certain extent, a subjective step. The approach adopted by WAAS has been to form a panel of experts on different components of the error and develop the threat model in a cycle of analysis, simulation, and real data analysis [2]. Important outcomes of this process have been:

- the ionospheric threat model [7],[8]
- the signal deformation threat model [9]
- Code noise and multipath (CNMP) curve [10]
- Code Carrier Coherence [9]
- Clock and ephemeris [11]
- Tropospheric delay [12]

Threat models limit the worst case behavior of the range errors and assign a certain probability to these events. Although they are based on data, threat models extrapolate real data through assertions based on physical mechanisms. These assertions need to be conservative and be compatible with the data.

From range error to position error

In the position domain, the difficulty is to find the right range domain characterization that propagates into a safe position error bound. The Protection Level equation adopted by SBAS (see below) [12] treats the range errors as if they were independent and zero mean. First, these errors might not be independent, it is therefore necessary to evaluate their correlation and make sure that the final position error bound is adequate. Among the reasons for the correlated errors there are:

- the fact that ionospheric corrections for different pseudoranges are computed using the same ionospheric pierce points
- the fact that the reference stations are synchronized using the satellites that are being corrected

It was said earlier that it was problematic to find the mean of the error distributions. It is however imperative to consider it when computing the convolution of these errors as the biases could add coherently.

In WAAS these threats are handled through simulation and worst case scenarios. WAAS uses several techniques to evaluate this threat. Among others, we can cite:

- Moment bounding [13]
- Gaussian bounding [5]
- Excess mass bounding [4],[14]

All of these techniques insure that biases and correlation in the pseudorange errors will be taken into account in the final position error bound computed by the user. Unfortunately, they also are very conservative which damages performance (see below). In the following sections we analyze with more detail the effects of biases and correlation in the computation of the position error bound.

EFFECT OF BIASES ON INFLATION

In this section we provide an example that illustrates the effects of the biases on the Protection Level. Assume that the SBAS user has n range measurements. Each range error measurement i is characterized by a Gaussian with variance σ_i^2. Let us also assume that each measurement might have an unobservable bounded bias b_i of any sign such that:

$$|b_i| \le \beta\sigma_i$$

For the purposes of this example, we have assumed that the factor β is the same for each range error. The vertical protection level VPL for a given user should be such that:

$$K\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_i^2} + \beta\sum_{i=1}^{n}\left|s_{i,vert}\right|\sigma_i \le VPL \qquad (1)$$

In Equation (1), $s_{i,vert}$ is the set of coefficients that projects the range measurements on the vertical position estimate. It is also the set of coefficients that projects the range errors onto the vertical position error. With the current SBAS standards [12], the form of the VPL is:

$$VPL = K\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_{\inf,i}^2} \qquad (2)$$

In Equation (2) $\sigma_{\inf,i}^2$ is inflated such that the inequality above holds for all users:

$$\sigma_{\inf,i} = \alpha\sigma_i$$

For simplicity, we will force the inflation to be the same for all measurements. This is a reasonable simplification because the parameter β is assumed to be the same for all measurements. The requirement on α is therefore:

$$K\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_i^2} + \beta\sum_{i=1}^{n}\left|s_{i,vert}\right|\sigma_i \le \alpha K\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_i^2}$$

It is easy to show that:

$$1 \le \frac{\sum_{i=1}^{n}\left|s_{i,vert}\right|\sigma_i}{\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_i^2}} \le \sqrt{n}$$

Although we will not show it here, for a given satellite geometry, there are users for which the first equality is almost true (users that weigh one satellite much more than the others) and users for which the second almost holds (users that weigh the satellites proportionally to the inverse of the standard deviation). The worst case user will have a set of coefficients such that:

$$\frac{\sum_{i=1}^{n}\left|s_{i,vert}\right|\sigma_i}{\sqrt{\sum_{i=1}^{n} s_{i,vert}^2\sigma_i^2}} \simeq \sqrt{n}$$

Therefore, we need:

$$1 + \frac{\beta}{K}\sqrt{n} \le \alpha$$

However, for many users, the ratio above is closer to unity. The inflation penalty resulting from protecting the worst case user compared to the best case user is given by:

$$\text{inflation penalty} = .51\beta$$

with K = 5.33 and a maximum of 14 satellites (12 GPS and 2 WAAS geostationary satellites). For example, if we need to protect for a bias with a magnitude equivalent to the half a standard deviation (β = .5), a relatively mild bias, the inflation penalty is 25%, which is large. This minimum inflation penalty is independent of any technique used to compute it.

EFFECT OF WORST CASE COVARIANCE AND LINK TO BIAS EFFECTS

In this section, the common features of biases and worse case correlation are shown. Let us suppose that part of the error is correlated, and that we would like to account for the worst case correlation of these errors. Let C be the covariance of the errors:

$$C = C_{corr} + C_{ind}$$

In this equation C_{ind} is the independent noise and C_{corr} the correlated noise. Let us suppose further that it is difficult to characterize the off-diagonal elements of C_{corr}, or that we are not able to transmit them to the users. The user must therefore assume the worst case correlation. In the following paragraphs we give an expression of the corresponding worst case position error standard deviation. $\sigma_{i,corr}^2$ is the variance of the i^{th} correlated range error.

The error variance due to the correlated error is given by:

$$\sigma_{v,corr}^2 = s_{vert}^T C_{corr} s_{vert} = \sum_{i=1}^{n} s_{i,vert}^2 C_{corr,ii} + 2\sum_{i<j} s_{i,vert} s_{j,vert} C_{corr,ij}$$

We want to solve the problem:

$$\text{Maximize } \sigma_{v,corr}^2 = \sum_{i=1}^{n} s_{i,vert}^2 C_{corr,ii} + 2\sum_{i<j} s_{i,vert} s_{j,vert} C_{corr,ij}$$

$$\text{Subject to } C_{corr,ii} = \sigma_{corr,i}^2 \text{ and } C_{corr} \geq 0$$

The last inequality means that C_{corr} must be positive definite. This fact implies that we have the inequality:

$$\left|C_{corr,ij}\right| \leq \sqrt{C_{corr,ii}}\sqrt{C_{corr,jj}}$$

This provides us an upper bound of the previous maximization problem:

$$\sigma_v^2 \leq \sum_{i=1}^{n} s_{i,vert}^2 C_{corr,ii} + 2\sum_{i<j} \left|s_{i,vert} s_{j,vert}\right| \sqrt{C_{corr,ii}}\sqrt{C_{corr,jj}} = \left(\sum_{i=1}^{n} \left|s_{i,vert}\right| \sigma_{corr,i}\right)^2$$

This upper bound can be reached taking C_{corr} defined by:

$$C_{corr} = \begin{bmatrix} \sigma_{corr,1} sign\left(s_{1,vert}\right) \\ \vdots \\ \sigma_{corr,n} sign\left(s_{n,vert}\right) \end{bmatrix} \begin{bmatrix} \sigma_{corr,1} sign\left(s_{1,vert}\right) \\ \vdots \\ \sigma_{corr,n} sign\left(s_{n,vert}\right) \end{bmatrix}^T$$

We therefore have:

$$\sigma_{vert}^2 = \sigma_{v,corr}^2 + \sigma_{v,ind}^2 \leq \left(\sigma_{v,corr} + \sigma_{v,ind}\right)^2$$

Finally:

$$\sigma_v \leq \sum_{i=1}^{n} \left|s_{i,vert}\right| \sigma_{corr,i} + \sigma_{v,ind}$$

This form is identical to Equation (1) which suggests that it is possible to use a bias term in the Protection Level equation to protect against small correlation in the pseudorange errors.

POSSIBLE MODIFICATIONS OF THE PROTECTION LEVEL EQUATION

In this section, we examine how the Protection Level equation could be modified to mitigate some of the difficulties in the proof of safety. In the previous sections we have identified three difficulties in the proof of safety: overbounding the tails of the error distribution, taking into account nominal biases in the pseudorange errors, and taking into account correlation between pseudorange errors.

Mitigating the difficulties due to biases and correlation

The obvious way to mitigate the effects of inflation due to biases is to add biases explicitly in the Protection Level equation:

$$VPL = K\sqrt{\sum_{i=1}^{n} s_{i,vert}\bar{\sigma}_i^2} + \sum_{i=1}^{n} \left|s_{i,vert}\right| b_i$$

Also as we saw in the previous section, it is also a way to account for worst case covariance.

Relaxing the overbounding requirement

In order to limit the overbounding requirement, it is possible to make the final error bound robust to the any one extreme error. This can be done by not trusting all measurements simultaneously (in the error bound). There are a multitude of techniques available to do that, including all the RAIM techniques with a provable rate of HMI. A simple way of achieving this is given by the following Protection Level Equation:

$$VPL = \max_i \left(\left|x_{\text{all in view}} - x_{\hat{i}}\right| + K\sigma_{vert,\hat{i}}\right) \tag{3}$$

In Equation (3) $x_{\text{all in view}}$ is computed using all available satellites, $x_{\hat{i}}$ and $\sigma_{vert,\hat{i}}$ are computed using the subset of n-1 satellites where the i^{th} satellite is excluded (A similar equation could be used for the HPL). The factor K is adjusted so that the PHMI requirement is met. Although, the VPL above is not predictable, it is possible to compute a predicted VPL by replacing the term:

$$\left|x_{\text{all in view}} - x_{\hat{i}}\right|$$

with the term:

$$K_{FA}\sqrt{\text{var}\left(x_{\text{all in view}} - x_{\hat{i}}\right)}$$

which can be computed ahead of time without the measurements. The factor K_{FA} is determined by the continuity requirement. For more information on this method, please refer to [15]. It is clear that such a Protection Level equation requires redundant measurements. This redundancy is not always available in the current constellation. However, future constellations (GPS III and Galileo) are likely to offer stronger constellations, making this approach more feasible.

A new equation may require more complex processing at the receiver and slight increase in bandwidth. Also, since it is possible to provide safety-of-life with the current MOPS in SBAS, while meeting LPV requirements it may not be clear to everyone what the benefit is. However, the proposed changes suggested in the previous sections are not intended for current single frequency SBAS. They are proposed here for future systems for which new standards will need to be developed, and where it is now possible to define them.

CONCLUSION

The PHMI needs to be clearly defined, in particular what the probability is conditioned on. It is important to have this definition in mind in the proof of safety, as it determines how the real data can be aggregated into meaningful empirical distributions. There are several challenges in the proof of safety in SBAS:

- overbounding the tails of the pseudorange errors
- accounting for nominal biases and correlation in the pseudorange errors

In WAAS the first problem is approached by using threat models, which limit the worst case behavior and are based on the observed data and analysis. The second problem is a mathematical one. There are several techniques allowing to account for the effect biases and correlation have on the position error.

Some of the challenges in the proof of safety could be greatly mitigated through changes in the system design. In particular, there are possible changes in the Protection Level equation that could:

- allow the relaxation of the requirements on the overbounding of the tails
- alleviate the effects of biases and correlation on performance

These changes should be considered and studied for future message standards, as they offer the possibility of increased performance and an easier proof of safety.

ACKNOWLEDGEMENTS

This work was sponsored by the FAA GPS Satellite Product Team (AND-730).

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